Grid-free Compressive Beamforming Using a Single Moving Sensor of Known Trajectory

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***Abstract*—Recently, the grid-free compressive sensing (GFCS) approach was proposed to perform direction of arrival (DOA) estimation of sources. With the advancement of estimation techniques using a single sensor with a known trajectory, it is proposed that a GFCS method can be extended to achieve grid-free two-dimensional localization. Through the trajectory of the sensor, the proposed approach extracts the spatial information by first reformulating the single-channel signal into multiple waveforms, where each group of consecutive waveforms satisfying the quasi-stationary condition can be constructed into a virtual array called the sub one sensor array (SOSA). The DOA of the source with respect to each SOSA is then estimated with GFCS. Accordingly, the final location of the source is computed as the point that minimizes the mean square distance to all DOA lines. Numerical and experimental results demonstrate that the proposed approach is able to perform grid-free localization of a sound source.**

Keywords—sensor array, localization, single sensor, linear motion, synthetic aperture.

I. Introduction

Traditionally, the direction of arrival estimation (DOA) of the sound source is achieved through estimation of the finite sound source observation using sensor arrays. Numerous algorithms for its DOA estimation have been derived under the condition that the array is static and non-varying. These include the conventional delay and sum [1], correlation-based Capon [2], and subspace-based approaches, including the multiple signal classifier (MUSIC) [3] and estimation of signal parameters via rotational invariance technique (ESPRIT) [4]. These have been derived based on the various signal and noise assumptions to obtain an optimal solution [5].

Recently, there has been much interest in solving such problem using compressive sensing (CS).The main difference between CS and the previous approaches is that the solution of a CS problem produces results that promote sparse solutions. Hence, solving the DOA estimation problem using CS results in high-resolution and robust acoustic images that outperform the traditional algorithms [6].

However, the CS approach has disadvantages. One key problem is the issue of basis mismatch [7]. Basis mismatch occurs when the direction of the source does not correspond to the current dictionary matrix, leading to spectral leakage and inaccurate reconstruction. One immediate solution is to employ a finer grid, at the expense of increased computational complexity, to ease the mismatch [7]. Despite this issue, because of the nature of CS, the dictionary matrix with a finer grid cannot be increased indefinitely. It needs to possess the restricted isometry property to ensure that a unique solution can be determined [8].

To overcome the basic mismatch problem, Xenaki and Gerstoft [9] proposed the grid-free compressive beamforming approach that moved away from the necessity of a dictionary matrix and solved the DOA problem by casting it as a polynomial equation. As such, the solution can be determined through polynomial rooting. This approach was shown to produce robust and high-resolution reconstruction, even with a non-uniform sensor array. However, because of the linear model requirement, the approach is limited to only DOA estimation.

Until recently, a great deal of research has been conducted on CS DOA for static arrays, but few have considered the possibility of extending the CS DOA problem to a single sensor. In terms of single sensor processing, previously, Hioka and Kishida [10, 11] proposed the CAROUSEL (CirculARly mOving sensor for USE of moduLation effect) architecture, which demonstrated that the DOA estimate could be achieved using a single sensor moving along the circumference of a circle. This approach was further investigated in Ang et al. [10], who proposed the one sensor array (OSA) approach to perform localization using a single moving sensor by deriving and applying a temporal correction factor to construct a synthetic aperture.

This paper proposes combining the concept of grid-free DOA and OSA to achieve a double extension: First, by taking advantage of a moving sensor, the original grid-free DOA is extendable to achieve a two-dimensional grid-free localization. Second, by using the grid-free DOA, the OSA localization is now solvable beyond using traditional approach. The key idea is to map the near-field localization problem into a multiple sub one sensor array (SOSA) DOA problem that can be solved by the higher- resolution grid-free DOA. This is possible because of the SOSA ability to produce a virtual array with any length, as long as the quasi-stationary [11] condition is achieved. After the DOA estimation with the SOSA, localization of the source can be achieved through a triangulation process that takes into account all the DOA estimates of the SOSA. Section II first presents an introduction to the signal model and the proposed approach to reformulating the single channel into a multi-channel waveform. Next, a temporal correction factor is derived from the multi-channel waveforms to extract the embedded spatial information and is applied to approximate the recording as a modified GFCS problem. Finally, an approach is presented to extend the estimated DOAs to source localization by triangulation of the estimated DOAs. This approach is named the SOSA approach. Section III includes numerical analyses conducted to illustrate the concept and performance of the proposed approach. Section IV presents the experiment conducted with the sensor mounted on an electric vehicle (EV) and the analyses to validate the proposed SOSA approach. Section V presents the conclusions of the paper.

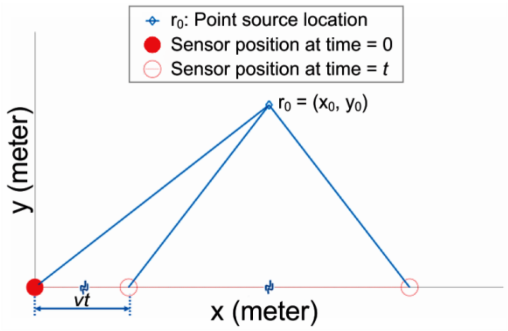

Fig. 1: Moving sensor configuration.

II. Problem Formulation

A. Signal Model for Single Acoustic Monopole

Consider a stationary point source signal captured by a single moving sensor as shown in Fig. 1. The source located at the spatial position $\mathbf{r} = (x, y)$ is denoted as

$$s(t) = A_s e^{j(\omega t + \psi)}, \quad (1)$$

where A_s, ω, and ψ are the amplitude, angular frequency, and phase of the point source, respectively. Because of the stationary assumption, A_s and ψ are assumed time-invariant. This signal is recorded by the moving sensor $p(t)$, initially located at the origin $(x, y) = (0,0)$, and is expressed as

$$p(t) = s(t) e^{-j\frac{\omega}{c} r(t)} + q(t), \;\; 0 \le t \le T, \quad (2)$$

where $r(t) = \|\mathbf{r} + \boldsymbol{v}t\|_2$ is the Euclidean distance between the source and sensor at time t moving with velocity $\boldsymbol{v}$; $q(t)$ is the additive white Gaussian noise used to model the spatial and temporal noise inherent in the moving sensor; and ω/c is the wavenumber of the source signal as the sensor moves within the recording window of T second. In addition, $e^{j\omega r(t)/c}$ constitutes the phase shift because of the propagation of the signal toward the moving sensor. The collected samples can now be decomposed into N frames, such that each frame is quasi-stationary [13]. The nth frame can be expressed as

$$p_n(t) = p(t + (n-1)\,\Delta T), \quad 0 \le t \le \Delta T\,, \quad (3)$$

where $n \in [1, N]$. Note that N is designed such that $\Delta T = T/N$ satisfies the quasi-stationary condition [11]. Each nth frame can be further decomposed into M multi-channels, with the mth channel modelled as

$$p_{m,n}(t) = p_n(\mathrm{t} + (m-1)\,\Delta \mathrm{T_M}), \;\; 0 \le \mathrm{t} \le \Delta \mathrm{T_M}\,, \quad (4)$$

where $m \in [1, M]$, $\Delta \mathrm{T_M} = \Delta T/M$. Now, the nth SOSA can be formed with M multi-channels, expressed compactly as

$$\boldsymbol{p}_n(t) = [p_{1,n}(t) \;\; \cdots \;\; p_{M,n}(t)]^T, \;\; 0 \le \mathrm{t} \le \Delta \mathrm{T_M}\,, \quad (5)$$

where $(.)^T$ denotes the transpose operation.

B. Design of Temporal Correction Factor

Let $t_n = \{(n-1)\Delta T \le t \le n\Delta T\}$, and removed $q(t)$ from the signal model in (5) for brevity, arrives at

$$\boldsymbol{p}_n(t) = \begin{bmatrix} s(t + 0\,\Delta T_M) e^{-j\frac{\omega}{c} r(t + 0\,\Delta \mathrm{T}_M)} \\ \vdots \\ s(t + (M-1)\,\Delta T_M) e^{-j\frac{\omega}{c} r(t + (M-1)\,\Delta \mathrm{T}_M)} \end{bmatrix}, \quad (6)$$

for all $t \in t_n$. Rearranging (6) yields the following

$$\boldsymbol{p}_n(t) = \underbrace{A_s e^{j(\omega t + \psi)}}_{\text{Source Component}} \underbrace{\begin{bmatrix} e^{j\omega\, 0\,\Delta \mathrm{T_M}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{j\omega\,(M-1)\,\Delta \mathrm{T_M}} \end{bmatrix}}_{\text{Temporal Delay}} \underbrace{\begin{bmatrix} e^{-j\frac{\omega}{c} r(t + 0\,\Delta \mathrm{T_M})} \\ \vdots \\ e^{-j\frac{\omega}{c} r(t + (M-1)\,\Delta \mathrm{T_M})} \end{bmatrix}}_{\text{Source Propagation}}, \quad (7)$$

From (7), it is now evident that $\boldsymbol{p}_n(t)$ consists of a temporal-delay component, $e^{j\omega\,(m-1)\,\Delta \mathrm{T_M}}$, and its source with propagation component $e^{j\omega t - j\frac{\omega}{c} r(t + (m-1)\,\Delta \mathrm{T_M})}$. Here, two observations can be noted. First, if the nth $r(t + (n-1)\,\Delta \mathrm{T_M})$ in the source with propagation component is approximately constant, it is similar to the nth sensor of a static array response. Second, the observed temporal-delay component in (7) is correctable with

$$\boldsymbol{A}_c = diag([e^{-j\omega\, 0\,\Delta T_M} \;\; \cdots \;\; e^{-j\omega\,(M-1)\,\Delta T_M}]), \quad (8)$$

a temporal correction factor. Applying the first observation and the temporal correction factor $\boldsymbol{A}_c$ arrives at the following expression for the nth SOSA

$$\boldsymbol{p}_{SOSA}^{(n)}(t) = \boldsymbol{A}_c \boldsymbol{p}_n(t) = e^{j\omega t} \begin{bmatrix} e^{-j\frac{\omega}{c} r(t + 0\,\Delta \mathrm{T_M})} \\ \vdots \\ e^{-j\frac{\omega}{c} r(t + (M-1)\,\Delta \mathrm{T_M})} \end{bmatrix}, \quad (9)$$

which has a similar response to a static array.

C. Grid-Free Compressive Beamforming

In this section, it is shown that DOA estimation of (7) can be achieved using the modified grid-free compressive beamforming method. First, consider the mth synthetic sensor sampled recording with $l = t \cdot F_s$, for all $t \in t_n$, where F_s is the sampling frequency of the sensor. Then

$$p_{m,n}(l) = A_s e^{j(\omega l + \psi)} e^{j\omega\,(m-1)\,\Delta \mathrm{T_M}} e^{-j\frac{\omega}{c} r(l + (m-1)\,\Delta \mathrm{T_M})}. \quad (10)$$

Using the fast Fourier transform (FFT), (10) can be represented in the complex domain as a sparse signal representation that is tractable and uses all the data samples coherently as

$$\boldsymbol{p}_{m,n}(k) = \tilde{A}_s a_{c_m}^* \sum_{l=0}^{L-1} e^{j(\omega-\omega_k)l}\, e^{-j\frac{\omega}{c}r(l+(m-1)\Delta\mathrm{T_M})}, \quad (11)$$

where $\tilde{A}_s = A_s e^{j\psi}$ is the complex signal amplitude that captures the signal power and its offset, $a_{c_m} = e^{-j\omega(m-1)\Delta \mathrm{T}_M}$, is the mth diagonal element of (8). Now, concatenating all the M complex reading arrives at

$$\boldsymbol{y}_n(k) = [\boldsymbol{p}_{1,n}(k) \quad \cdots \quad \boldsymbol{p}_{M,n}(k)]^T. \quad (12)$$

Because of the nature of the SOSA formulation, the SOSA can be always constructed such that signal with wavelength λ is located in the far-field region relative to D_n, the aperture of the nth SOSA (i.e., $2D_n^2/\lambda \ll r(t), \forall\, t \in t_n$ [1]). Accordingly, the mth sensor of (12) can be approximated as

$$\boldsymbol{y}_{n,m} \cong a_{c_m}^* x_n e^{j\frac{\omega}{c}d(m-1)\cos\theta}, \quad (13)$$

where x_n is the complex amplitude of the source, d is the inter sensor spacing between each m th sensor, and the measurement vector of the nth SOSA is

$$\boldsymbol{y}_n \cong \boldsymbol{A}_c^H \boldsymbol{\mathcal{F}}_M x, \quad (14)$$

where $\boldsymbol{\mathcal{F}}_M$ is a linear operator (inverse Fourier transform) that maps the signal x into $\boldsymbol{y}_n$. Now, by multiplying (14) with (8)

$$\bar{\boldsymbol{y}}_n \cong \boldsymbol{A}_c \boldsymbol{y}_n = \boldsymbol{\mathcal{F}}_M x. \quad (15)$$

The standard atomic norm minimization form of the grid-free compressive beamforming is arrived at in (15), which can be cast as a semi-definite programming (SDP) problem as in [9]. Concretely, the SDP problem can be written as

$$\max_{\boldsymbol{c}_n, \boldsymbol{Q}} Re(\boldsymbol{c}_n^H \bar{\boldsymbol{y}}_n) - \epsilon\|\boldsymbol{c}_n\|_2 \;\; Subject\ to \begin{bmatrix} \boldsymbol{Q}_{M\times M} & \boldsymbol{c}_{M\times 1} \\ \boldsymbol{c}_{1\times M}^H & 1 \end{bmatrix} \succcurlyeq 0$$

$$\sum_{i=1}^{M-j} \boldsymbol{Q}_{i,i+j} = \begin{cases} 1, & j = 0 \\ 0, & j = 1,\cdots, M-1 \end{cases}, \quad (16)$$

where ϵ is the bounded value of the noise and potential error in the far-field approximation. Finally, DOA estimation can be solved in three steps:

a) Acquire sets of complex coefficients by solving the SDP problem.
b) Identify the support of the complex coefficients that lies on the unit circle.
c) Find the angle of that support.

D. Localization

After the estimation of N DOA, localization is now achieved by triangulating all angles to create an intersection between all the estimates. This is performed by first expressing the N DOA as equation of line span by the DOA.

$$\alpha_n x + \beta_n y + \gamma_n = 0, \quad (17)$$

where α_n, β_n, and γ_n are the coefficients describing the line span by nth DOA. Next, the estimated source location $\hat{\mathbf{r}} = (\hat{x}, \hat{y})$ can be found as the point that minimizes all the squared distance, $d_1^2, \dots, d_N^2$, between all the N DOA lines and the point $\hat{\mathbf{r}}$. Mathematically, this is expressed as

$$\min_{x,y} \sum_{n=1}^N w_n^2 d_n^2 = \min_{x,y} \sum_{n=1}^N \frac{w_n^2(\alpha_n x + \beta_n y + \gamma_n)^2}{\alpha_n^2 + \beta_n^2}, \quad (18)$$

where w_n is used to set the contribution of the nth DOA line. Finally, the solution of (18) has an analytical expression of

$$\hat{x} = -\frac{(\mathrm{B\Gamma - ZC})}{\mathrm{B^2 - AZ}},\ \hat{y} = +\frac{(\mathrm{A\Gamma - BC})}{\mathrm{B^2 - AZ}}, \quad (19)$$

where

$$\mathrm{A} = \sum k_n \alpha_n^2,\ \mathrm{B} = \sum k_n \alpha_n \beta_n,\ \mathrm{C} = \sum k_n \alpha_n \gamma_n,\ \mathrm{Z} = \sum k_n \beta_n^2,\ \Gamma = \sum k_n \beta_n \gamma_n,\ \ k_n = \frac{w_n^2}{\alpha_n^2 + \beta_n^2}.$$

Here, it is prudent to state that the solution to (18) was determined to have the form of the weighted least square when expressed as

$$\min \|\mathbf{WEr} - \mathbf{W\gamma}\|_2, \quad (20)$$

where $= \mathrm{diag}(k_1^{0.5}, \dots, k_N^{0.5})$, $\hat{\mathbf{r}} = [\hat{x}, \hat{y}]^{\mathrm{T}}$ is the estimated source coordinate, $\mathbf{E} = [\boldsymbol{\alpha}, \boldsymbol{\beta}]$, is the concatenate of the $[\alpha_1, \cdots, \alpha_N]^{\mathrm{T}}$ and $[\beta_1, \cdots, \beta_N]^{\mathrm{T}}$ vector, respectively, $\boldsymbol{\gamma} = [\gamma_1, \cdots, \gamma_N]^{\mathrm{T}}$.

III. Numerical Analysis

This section presents numerical simulations to illustrate the validity of the SOSA approach established in Section II. In the simulation, a moving sensor initially located at the origin of the Cartesian coordinate system is made to move with a velocity of $\boldsymbol{v} = (v_x, v_y) = [1.4, 0]$ m/s in the $+x$ direction and a sampling rate of 25.6 kHz. A source was placed at location $\mathbf{r} = [30,15]$ m with a signal-to-noise (SNR) ratio of 0 dB. The acoustic propagation medium is assumed to be air with $c = 343$ m/s.

Fig. 2 depicts the plot of the SOSA localization with (19) and (20) using the proposed GFCS-SOSA approach. Here, the SOSA is constructed using 28 sec of recording. With each frame having $\Delta T = 1$ sec, a total of $N = 25$ SOSA is constructed. Each SOSA was then modelled with $M = 10$ synthetic sensor with $\Delta T_M = 0.1$ sec, Fourier transformed with 2^{11} samples, and $\epsilon = 1$. Here, it is evident that the proposed GFCS-SOSA is able to localize effectively.

IV. Experimental Results

After the simulation, the SOSA algorithm was applied to a set of data collected from a driving experiment from [12], as depicted in Fig. 3. The microphone is mounted on top of an electric vehicle (EV) located at $(x, y) = (+6.605, -0.25)$ m. The EV traverses along a linear path at approximately $v = 5$ km/hr with a speaker located at $\boldsymbol{r}_0 = (30, 15)$ broadcasting a square wave of 250 Hz. Similar to the numerical analysis, the recording was performed for a duration of $\Delta T = 28$ sec at a sampling rate of $F_s = 25.6$ kHz for, a total of $N = 25$ SOSA with each n th SOSA having $M = 10$ synthetic sensors, Fourier transformed with 2^{11} samples, and $\epsilon = 1$.

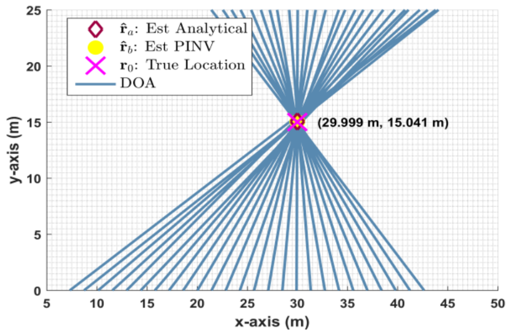

Fig. 2: GFCS-SOSA localization on the simulated moving sensor recording, $\hat{\boldsymbol{r}}_l = (29.999, 15.041)\ m$.

Fig. 4 shows the DOA localization using the GFCS beamforming. Although the source was localized with a source location estimation error, it is attributable to the difficulty of maintaining a constant velocity along the drive-by trajectory [12]. In essence, the results demonstrated that SOSA localization of an acoustic source is achievable using the SOSA approach with the GFCS and an approximately known velocity.

V. Conclusion

This paper addresses the problems of performing localization when there is insufficient space for sensor placement and demonstrates that source localization is achievable using the SOSA approach based on a single moving sensor of a known piecewise-linear trajectory. The proposed approach consists of three steps: (1) The data are acquired from a single sensor moving along a trajectory with a known velocity $\boldsymbol{v}$ while observing the signal; (2) the data are preprocessed to decompose the signal into multiple synthetic subarrays, where each of the subarrays is synchronized through the application of a temporal correction factor; (3) multiple DOA estimations are performed before combining the results to triangulate the source location. The numerical simulation conducted and experimental verification shows that the proposed technique can be applied to localize an acoustic source using a single moving sensor. Essentially, this work advances the framework of using one sensor for DOA estimation beyond a circular trajectory and GFCS for localization using comparable resources.

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Fig. 3: Experimental setup of the OSA recording using the EV with the microphone mounted on top.

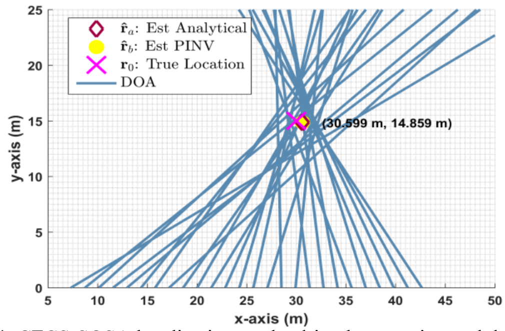

Fig. 4: GFCS-SOSA localization on the drive-by experimental data, $\hat{\boldsymbol{r}}_l = (30.599, 14.859)\ m$

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