Detection of Meaningful Line Segment Configurations

Jierong Cheng, Wei Xiong, Jia Du, Wenyu Chen, and Ying Gu

Institute for Infocomm Research, Agency for Science, Technology and Research, Singapore

Abstract—**We proposed a method for the detection of meaningful line segment configurations, based on the *a contrario* framework. The method can detect complete or incomplete regular polygons from a set of line segments. We define the strength of a configuration based on its resemblance to regular polygons. A configuration is meaningful if the expectation of such occurrence would be very small in an image of random line segments. The method is tested on both synthetic images and real images and detects 92.8% of the polygon edges.**

I. Introduction

In all kinds of image analysis tasks, edge detection and object recognition are among the most fundamental and essential ones. Commonly, these tasks involve the minimizing of an energy functional such as the Mumford-Shah model [1], or decision making which maximizes a probability function such as the Bayesian model [2]. While these methods can always find a segmentation of the image, they do not check whether the segmentation is meaningful or not [3]. Another drawback of all variational frameworks is that their performance is very sensitive to the parameter setting.

Desolneux et al. proposed an *a contrario* method for detecting geometric structures in an image without any prior information [3]. The main idea is that an observed geometric event, be it an alignment of points or a spot, is "meaningful" if the expectation of its occurrences would be very small in a random image. A mathematical framework was built to quantify the Helmholtz principle that if the expectation of an observed configuration in an image is very small, then it makes sense to group these objects to form a 'Gestalt' (German: shape, form). Therefore, this method is proposed to make decision on the meaningfulness of a configuration of points, rather than proposing a new segmentation methods [3].

The original method is applied to the detection of alignments of points. Since then, the *a contrario* framework has been applied successfully to many other image analysis tasks. For instance, von Gioi et al. developed a popular line segment detector (LSD) [4] which works efficiently without the need of any parameter tuning. LSD finds line segments in an image by measuring the number of aligned pixels in a rectangle. Xia et al. also used *a contrario* method for validation during junction detection in natural images [5]. In our previous work [6], we proposed a hole crack detection method for defect inspection based on the *a contrario* method.

In this work, we proposed a method for the detection of meaningful line segment configurations, based on the *a contrario* framework. The method detects regular polygons from a set of line segments found by the LSD method [4]. Firstly, we define the strength of a line segment configuration based on their respective centroids and their lengths. A configuration is said to be ε-meaningful if and only if the number of false alarms (in an image containing line segments with random starting and ending points) associated to its strength is less than a constant ε. According to this definition, we developed an efficient algorithm for the search of such meaningful configurations starting from a minimum side number. Existing methods which detect polygons from line segments [7] or from point sets [8], depend on the availability of all polygonal sides or all vertices. In contrast, the strength of this work lies on the capability of detecting polygons with missing/corrupted sides in the images.

II. Methodology

A. Line segment detection and merging

First of all, line segments are detected from the original image using the LSD algorithm [4]. However, these line segments are often incomplete or overlapped each other. Therefore, to extract useful line segments which representing object boundaries, it is necessary to merge those who are collinear or overlapped.

The distance between two line segments l and l' is measured by [9]

$$D(l, l') = \sqrt{(\Delta\theta/\theta_\delta)^2 + (\Delta d/d_\delta)^2} \qquad (1)$$

where $\Delta\theta$ and Δd are the angle and the midpoint perpendicular distance between the two line segments, respectively. θ_δ and d_δ are thresholds for the angle and the perpendicular distance, respectively. Two line segments will be merged if $D \le 1$, which requires $\Delta\theta \le \theta_\delta$ and $\Delta d \le d_\delta$.

B. Line segment configuration

A regular polygon has all sides equal and all angles equal. Let L_i and θ_i be the length and the angle of line segment l_i. If l_i belongs to one of the M sides of a regular polygon, the centroid $c_i(x_i, y_i)$ of the polygon can be recovered by trigonometric functions as follows:

$$x_i = x_i^0 \pm H_i \cos(\theta_i + \pi/2) = x_i^0 + z_i H_i \cos(\theta_i + \pi/2)$$
$$y_i = y_i^0 \pm H_i \sin(\theta_i + \pi/2)$$
$$= y_i^0 + z_i H_i \sin(\theta_i + \pi/2) \qquad (2)$$

where (x_i^0, y_i^0) is the midpoint of l_i, $z_1, ..., z_n \in \{-1, 1\}$ are binary variables, and $H_i = L_i/(2\tan(\pi/M))$ is the apothem of the polygon. See Fig. 1 for illustration.
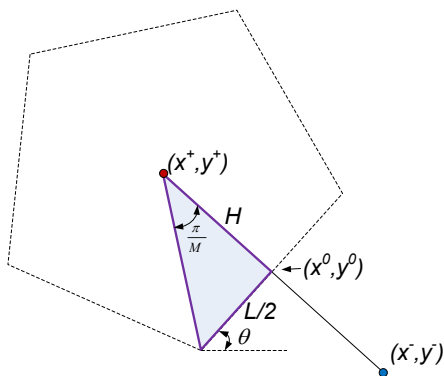
Fig. 1. Calculate the centroid of a pentagon from one of its side.

A set of n line segments $\{l_i | i = 1, 2, ...n\}$ comprises a *configuration* $c(l_1, l_2, ..., l_n)$. We define the strength of a configuration c as the expectation of line length $\mathbb{E}(L)$ divided by the sum of the variance of centroid locations and the variance of line segment lengths:

$$t_M(c) = \frac{\mathbb{E}(L)}{\Sigma_{i=1}^n \|c_i - \mu\|^2 / n + Var(L_i)}$$
$$= \frac{\mathbb{E}(L)}{Var(X_i) + Var(Y_i) + Var(L_i)} \quad (3)$$

where μ is the center of all n centroids and $Var(X_i)$ and $Var(Y_i)$ are the variance of x_i and $y_i, i = 1, 2, .., n$, respectively. The definition is based on the fact that if these line segments forms a regular polygon, they should be very close in both of their respective centroid locations and their lengths. $\mathbb{E}(L)$ is a normalization coefficient estimated from the image. For a synthetic image with its width and its height being 1 respectively, the expectation of the length of a randomly generated line segment within the image is 0.25. For real images, $\mathbb{E}(L)$ is estimated by the median of line segment lengths in the image.

However, since each line segment has two corresponding centroids on different sides ((x^+, y^+) and (x^-, y^-) in Fig.1), the computation of $t(c)$ is not so straightforward. It is an integer programming problem in which all n variables are restricted to be binary integers

$$t_M(c) =$$
$$\min_{z_1, ..., z_n \in \{-1, 1\}} \frac{\mathbb{E}(L)}{Var(X_i) + Var(Y_i) + Var(L_i)}. \quad (4)$$

C. Meaningful configuration detection

Now that the configuration strength is defined, we consider the event that a random configuration C of line segments in the *a contrario* model has larger strength than the observed configuration c. The expected number of such events or the number of false alarms (NFA) of c is defined as

$$\text{NFA}_M(c) = \#\mathcal{C} \cdot P_{\mathcal{H}_0}[t_M(C) > t_M(c)] \quad (5)$$
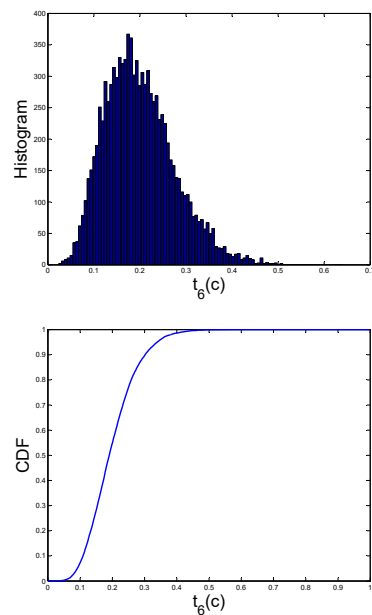

Fig. 2. (a) Histogram of t_M when $M = 6$ calculated from 10000 randomly generated line segments within an image with width and height of 1. (b) Cumulative distribution function calculated from (a).

where $\#\mathcal{C}$ is the total number of possible configurations being considered, $P_{\mathcal{H}_0}$ is the probability under a null hypothesis $\mathcal{H}_0$. A configuration is said to be ε-meaningful if and only if it satisfies $\text{NFA}(c) \leq \varepsilon$. The null hypothesis $\mathcal{H}_0$ is defined as

- $t_M(c)$ for all possible c in an image is made of independent random variables
- $t_M(c)$ follows a distribution which can be estimated from its histogram.

Let N be the total number of line segments detected by LSD algorithm. We compute $\#\mathcal{C}$ with respect to n line segments by

$$\#\mathcal{C}(M, N, n) = \binom{N}{M}\binom{M}{n}, n < M < N. \quad (6)$$

The distribution of $t_M(c)$ is estimated from its histogram computed from random generated line segments in an image. An example of the cumulative distribution function of $t_M(c)$ is shown in Fig. 2.

The maximal-meaningful side number for a configuration c is defined as

$$M^*(c) = \arg \min_{M_{\min} \leq M \leq M_{\max}} \text{NFA}_M(c). \quad (7)$$

For example, $M^*(c) = 4$ means c is more likely to be a square than any other shapes. The overall meaningful configuration detection procedure is described in Algorithm 1.

III. EXPERIMENTAL RESULT

In our experiments, the parameter settings are as follows: $\theta_\delta = 3°$, $d_\delta = 5$, $\varepsilon = 1$, $M_{\min} = 3$, $M_{\max} = 6$. In other words, the regular polygons to be detected from the image are triangles, squares, pentagons, and hexagons. To reduce the

Algorithm 1 Meaningful configuration detection

Require: A set of line segments $L = \{l_i | i = 1, 2, ... N\}$, and parameters ϵ, M_{min}, M_{max}

Ensure: A set of maximal ϵ-meaningful configurations C (if any).

1: Let C be an empty set of configuration candidates;
2: **for** each possible configuration c consisting of M_{min} line segments from L **do**
3:    Compute $NFA_M(c), M_{min} \leq M \leq M_{max}$ by using Eqn. (5);
4:    find the maximal-meaningful size number $m = M^*(c)$;
5:    **if** $NFA_m(c) \leq \epsilon$ **then**
6:       **if** $m = M_{min}$ **then** {forms a complete polygon}
7:          $C = C \cup c$
8:          $L = L \setminus c$
9:       **else if** $m > M_{min}$ **then** {find the remaining sides of the polygon}
10:          **for** k = 1 to $m - M_{min}$ **do**
11:             Find the one maximal-meaningful additional line segment $l^* = \arg\min_{l_i \in L} NFA_m(c \cup l_i)$;
12:             **if** $NFA_m(c \cup l^*) \leq \epsilon$ **then**
13:                $c = c \cup l^*$
14:                $L = L \setminus l^*$
15:             **else**
16:                break;
17:             **end if**
18:          **end for**{found all the remaining sides of the polygon}
19:          $C = C \cup c$
20:          $L = L \setminus c$
21:       **end if**
22:    **end if**
23: **end for**

computational time of Algorithm 1, we leave a configuration out of account if the width or the height of its bounding box exceeds $3*\mathbb{E}(L)$.

A synthetic image containing these four shapes is manually drawn (Fig. 3(a)). In addition to the 18 line segments detected from the image (Fig. 3(b)), 10 or 30 randomly generated line segments are added as well. Our algorithm successfully detects the four polygons from all three scenarios without any false alarm.

Our next experiment consists of 15 real images of various polygon objects such as bolts, traffic signs, and architectures. Six example images are shown in Fig.4. For most of the bolts, only 3 to 5 sides are among these line segments found by LSD followed by merging (Fig. 4(a)). The rest are missed due to similar intensities between the bolt and the background. The false positives are mainly caused by the edges detected at the bottom of the bolts. From the soccer ball image shown in Fig. 4(c), the black patch in the middle is detected as a pentagon; the surrounding white patches are not perfect hexagons due to perspective transform and are therefore not recognized. From the photo of an impossible triangle shown in Fig. 4(i), all five

(a)

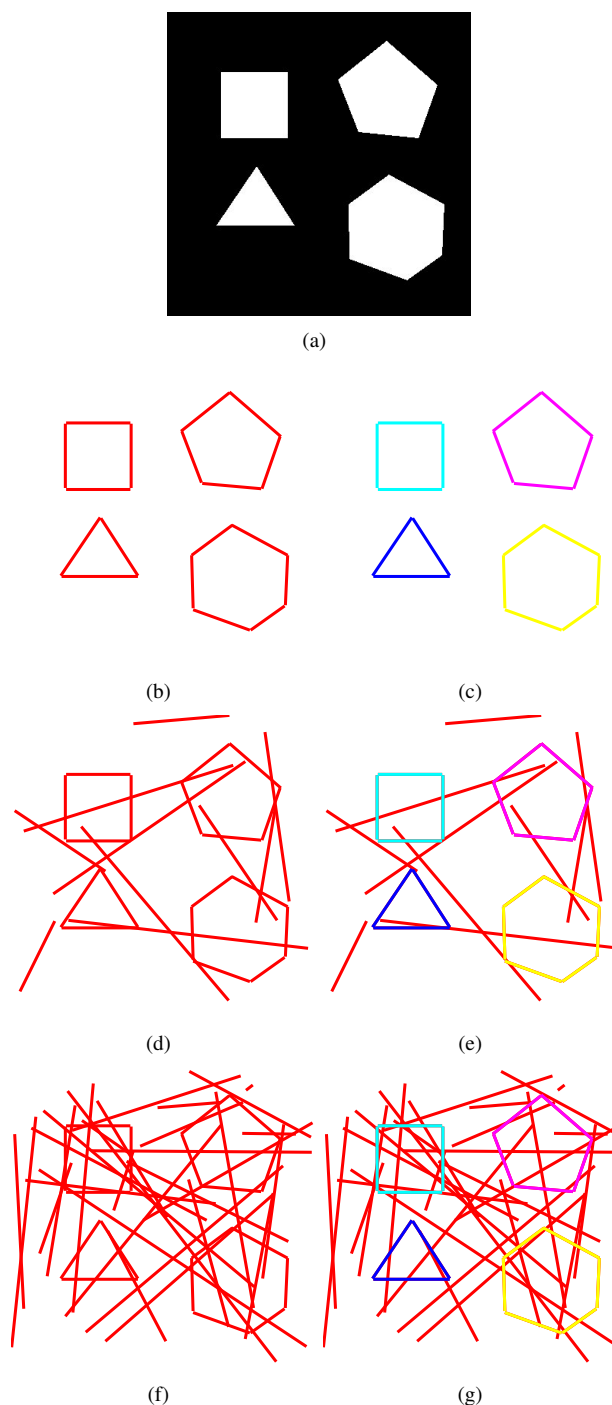(b)   (c)

(d)   (e)

(f)   (g)

Fig. 3. (a) Original hand-drawn image. (b) Line segments after applying LSD and merging. (c) Meaningful line segment configurations found from (b). (d) Line segments in (b) plus 10 randomly generated line segments. (e) Meaningful line segment configurations found from (d). (f) Line segments in (b) plus 30 randomly generated line segments. (g) Meaningful line segment configurations found from (f).

(a)   (b)

(c)   (d)

(e)   (f)

(g)   (h)
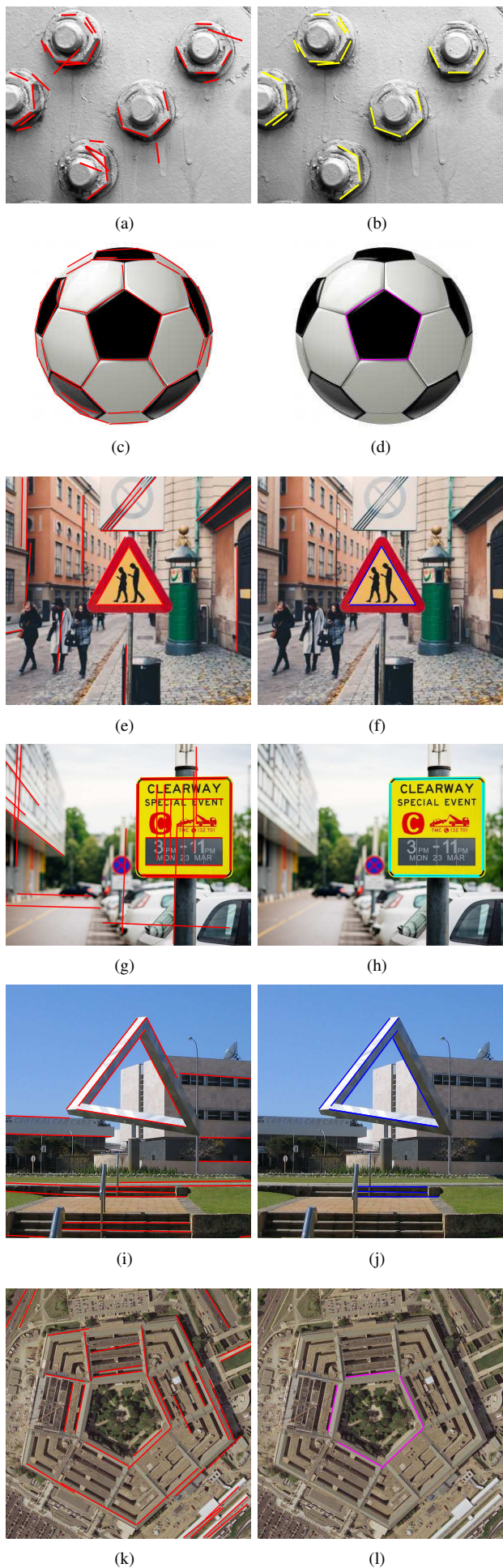
(i)   (j)

(k)   (l)

Fig. 4. First column: line segments after applying LSD and merging on a real image. Second column: meaningful line segment configurations found from the line segments which forms a regular polygon.

TABLE I
CONFUSION MATRIX FOR MEANINGFUL LINE SEGMENT DETECTION.

	Predicted: meaningful	Predicted: non-meaningful	
Actual: meaningful	TP = 128	FN = 10	138
Actual: non-meaningful	FP = 46	TN = 498	544
	174	508	91.8%

line segments belonging to the triangle are detected, while there are three false positives on the steps because of their similar length and orientation.

Out of the total 682 line segments found after applying LSD and the merging operation, 138 are sides in regular polygons, i.e., belonging to regular polygons. The proposed method detected 92.8% of sides of regular polygons, even when they don't form a complete polygon. The overall accuracy is 91.8% and the confusion matrix is given in Table I.

IV. CONCLUSION

We have proposed a method for the detection of meaningful line segment configurations, based on the *a contrario* framework. The strength of a configuration is formulated to measure its resemblance to regular polygons. Those configurations forming regular polygons are detected based on their meaningfulness. Specifically, the threshold on the configuration strength is determined automatically by requiring that the number of false alarms under a hypothesis for the image is less than 1. The experimental results tested on synthetic images and real images show that the proposed method performs well in detecting both complete and incomplete regular polygons. Our future work is to extend this method to other line segment configurations such as grids.

REFERENCES

[1] Luminita A. Vese and Tony F. Chan, "A multiphase level set framework for image segmentation using the mumford and shah model," *Int. J. Comput. Vision*, vol. 50, no. 3, pp. 271–293, Dec. 2002.

[2] Stuart Geman and Donald Geman, "Stochastic relaxation, gibbs distributions, and the bayesian restoration of images," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 6, no. 6, pp. 721–741, Nov. 1984.

[3] Agns Desolneux, Lionel Moisan, and Jean-Michel Morel, "Meaningful alignments," *International Journal of Computer Vision*, vol. 40, no. 1, pp. 7–23, 2000.

[4] Rafael Grompone von Gioi, Jérémie Jakubowicz, J.-M. Morel, and Gregory Randall, "Lsd: a line segment detector," *Image Processing Online*, Mar 2012.

[5] Gui-Song Xia, Julie Delon, and Yann Gousseau, "Accurate junction detection and characterization in natural images," *International Journal of Computer Vision*, vol. 106, no. 1, pp. 31–56, 2014.

[6] Jierong Cheng, Wei Xiong, Yue Wang, Shue Ching Chia, Wenyu Chen, Jia Du, Ying Gu, and Victor Ter Shen Kow, "Chord: Cascaded and a contrario method for hole crack detection," in *Proceedings of IEEE International Conference on Image Processing (ICIP)*, 2015, pp. 3300–3304.

[7] Alfredo Ferreira, Jr., Manuel J. Fonseca, and Joaquim A. Jorge, "Polygon detection from a set of lines," in *In Proceedings of 12 O ENCONTRO PORTUGUŁS DE COMPUTAO GRFICA (12th EPCG*, 2003, pp. 159–162.

[8] Greg Aloupis, Jean Cardinal, Sébastien Collette, John Iacono, and Stefan Langerman, "Detecting all regular polygons in a point set," *CoRR*, vol. abs/0908.2442, 2009.

[9] Youngwook Paul Kwon, "Line segment-based aerial image registration," M.S. thesis, EECS Department, University of California, Berkeley, May 2014.