# Colorization Algorithm based on Image Segmentation and Graph Signal Processing

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Abstract—This paper proposes a new colorization algorithm for converting grayscale image to color image by using several colored pixels. In order to obtain a properly colorized image, graph signal processing and image segmentation technique are introduced. After a graph is constructed on the given grayscale image, graph signal recovery technique based on graph Fourier transform recovers several important color information. The whole image is colorized by the Levin's algorithm. Experimental results using 5 images show 1.82dB to 10.2dB improvement in PSNR. The proposed algorithm also reduces the fading of color compared with the Levin's algorithm.

#### I. INTRODUCTION

Colorization is a technique for converting grayscale image to color image using several limited color information. There are two types of colorization techniques, i.e., automatic colorization technique[1][2] and user guided colorization technique[3][4][5]. The former has the disadvantage that the result often contains inaccurate colors. The latter approach can achieve impressive results if the user gives colors to each of the image regions of different colors. This paper improves latter colorization algorithm.

Levin's colorization method, in which color image is obtained as the solution of the optimization problem, assumes that the changing of chrominance values between adjacent pixels correlates the changing of the luminance values of its pixels. In order to improve the performance of the Levin's method, we have introduced the sparse optimization approach into image colorization[5]. However, the results are faded when less color information is given because color information is expanded to whole an image from the given color pixels and because color information can not be transmitted to a remote area.

Recently, the research of the graph signal processing has been improved to process the graph signal on the vertices of a given graph[6][7][8]. S.K.Narang, et al., proposed a graph signal interpolation method which recovers the unknown graph signals from limited graph signals[9].

This paper proposes a new colorization algorithm to obtain natural color image even when a few color pixels are given. In the algorithm, several pixels on the grayscale image are automatically selected using the image segmentation technique and color pixels are given by user. Both pixels are defined as the vertices of a graph and vertex are calculated based on luminance values and image segments. The chrominance values are set as a graph signal, and the graph signal interpolation method[9] is applied. Finally, Levin's colorization method colorizes the all pixels of an image.

# II. GRAPH SIGNAL RECOVERY USING GRAFH FOURIER TRANSFORM

The study of graph signal processing deals with signals called graph signals[6][7][8]. Let  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, W)$  denote an undirected connected weighted graph consisting of a finite set of vertices  $\mathcal{V}$  with  $|\mathcal{V}| = N$ , a set of edges  $\mathcal{E}$ , and a weighted adjacency matrix  $W \in \mathbb{R}^{N \times N}$  whose (i, j)th element  $w_{i,j}$  represents the weight of the edge e = (i, j) connecting vertices i and j. The degree matrix  $D = diag(d_1, d_2, \ldots, d_N)$  is a diagonal matrix, where  $d_i$  represents the sum of link-weights connected to *i*th vertex. Here, the combinatorial Laplacian matrix L is given as L = D-W, and the normalized Laplacian matrix is denoted as follows,

$$\mathcal{L} = D^{-1/2} L D^{-1/2}. \tag{1}$$

Let  $\lambda_i$  and  $u_{\lambda_i}$  (i = 0, ..., N-1) denote the eigenvalue and eigenvector of  $\mathcal{L}$ . *G* is defined as follows,

$$G = \left[ u_{\lambda_0}, \dots, u_{\lambda_{N-1}} \right].$$
<sup>(2)</sup>

 $\boldsymbol{g} \in R^N$  is defined as a graph signal, and graph Fourier transform is denoted as follows,

$$\boldsymbol{g} = \boldsymbol{G} \cdot \boldsymbol{s},\tag{3}$$

where  $s \in \mathbb{R}^N$  is a vector called graph spectrum. Furthermore, the following equation is satisfied,

$$\boldsymbol{s} = \boldsymbol{G}^T \cdot \boldsymbol{g}. \tag{4}$$

A method to estimate a whole graph signal from a part of given graph signal has been proposed in [9]. Therefore the study in [9] considers uncomplete graph signal , that is, a graph signal has two types of element that has a signal value and that is empty element. Here R is denoted as the set of the elements that the graph signal value is known. Let  $g_k \in R^{|R|}$  denote a sub vector of g constructed from the elements included in R, and let  $g_{uk} \in R^{N-|R|}$  denote a sub vector of g constructed from the elements included in R.

where  $R^c$  is a complementary set of R. Then graph spectrum  $\hat{s} \in R^b$  is calculated as follows,

$$\hat{\boldsymbol{s}} = \boldsymbol{G}_k \cdot \boldsymbol{g}_k. \tag{5}$$

Here  $G_k$  is a submatrix of  $G^T$  constructed from the (i, j)th element, where  $(i, j) = \{(i, j) | i \in \{1, ..., b\}, j \in R\}$  and b is a cutoff frequency proposed in [9]. Using this graph spectrum, the unknown graph signal value are estimated as follows,

$$\boldsymbol{g_{uk}} = \boldsymbol{G}_{uk} \cdot \hat{\boldsymbol{s}}.$$
 (6)

Here  $G_{uk}$  is a submatrix of G constructed from the (i, j)th element, where  $(i, j) = \{(i, j) | i \in \mathbb{R}^c, j = \{1, ..., b\}\}.$ 

# III. COLORIZATION BASED ON GRAPH SIGNAL PROCESSING

In order to obtain the color image from the given grayscale image with several color pixels, this paper proposes a novel colorization algorithm based on the graph signal processing.

First, the given grayscale image is segmented to N segments by using superpixel segmentation algorithm proposed in [10]. Let P and Q denote the set of the center pixels of each superpixel and the set of the given color pixels by a user, respectively, and V is denoted as  $V=P \cup Q$ . Here, M is denoted as the number of the given color pixels, that is, |Q| = M is satisfied. Previous colorization algorithm such as Levin's one can colorize the pixels whole an image appropriately when the chrominance values of the pixels included in V are given since Levin's colorization algorithm colorizes the image by expanding the color information from the known color pixels. However, set V includes the both pixels having chrominance values and pixels not having chrominance value. Therefore this paper proposes to apply the graph signal processing based signal recovery technique to estimate the chrominance values of the pixels in V.

In order to construct the graph recovering the chrominance values in V appropriately, this paper assumes that the chrominance values are similar when the pixels have similar luminance values and that the chrominance values are similar when the superpixels are similar shape. Then this paper proposes the method to formulate the shape of the superpixels.

Let  $S_m$  denote the number of the pixels located at the *m*th superpixel, and let  $r_m$  denote the maximum distance between the pixels in *m*th superpixel and the center pixel of *m*th superpixel. This paper proposes the following objective function to denote the shape of each superpixel,

$$T_m = \frac{S_m}{\pi r_m^2} \times S_m. \tag{7}$$

The first factor of the above function is a value correlated with the shape of superpixels, and the second factor of the above function represents the size of the superpixel. Therefore,  $T_m$ includes the both shape and size information, and the similar objects being discontinuous regions each other on an image such as a petal gives the similar values of  $T_m$ . Fig.1 shows the explanation of  $T_m$  and Fig.2 shows the advantage of  $T_m$ .

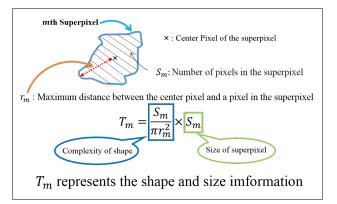


Fig. 1: explanation of  $T_m$ 

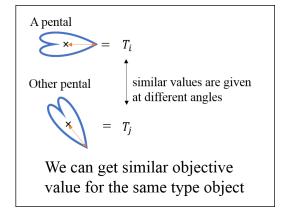


Fig. 2: Example of the effectiveness of  $T_m$ 

Based on the equation (7) and the luminance values of the pixels in V, this paper presents the weights of the between mth and nth vertex as follows,

$$w_{mn} = \exp(-((Y(m) - Y(n))^2)^{\frac{1}{2}}) + \alpha \times \exp(-((T(m) - T(n))^2)^{\frac{1}{2}}), \quad (8)$$

where Y(i) indicates the luminance value of the center pixel of the *i*th superpixel, and  $\alpha$  is a positive constant. This paper gives the graph signal g constructed as the chrominance values at the pixels in V, and then, we can use the known graph signal values  $g_k$  corresponding to the color pixels in Q. The graph Fourie transform matrix G is calculated by using the weights (7), and the estimated graph signal value is given based on the equations (4) and (5). Now, we can use the chrominance values in V to colorize the whole an image by Levin's colorization algorithm.

Unfortunately, the cutoff frequency proposed in [9] do not work well to estimate the chrominance values in P empirically. Thus, this paper proposes the algorithm to achieve appropriate cutoff based on the iterative approach. Let  $R_m$  (m = 1, ..., M)denote the set of the pixels in each superpixel including the pixel having in Q. Note that  $R_i$  corresponds to  $R_j$  when the

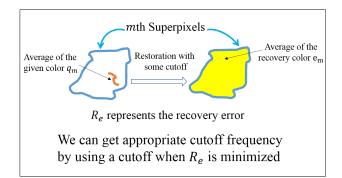


Fig. 3: Explanation of  $q_m$  and  $e_m$ 

one superpixel have multiplicate given color pixels. Here, we obtain a colorized image using a cutoff, and the sum of local recovery error is denoted as follows,

$$R_e = \sum_{m=1}^{M} |q_m - e_m|,$$
 (9)

where  $q_m$  is the average of the chrominance values given by a user in  $R_m$ , and  $e_m$  is the average value of the recovered chrominance values in  $R_m$ . Then this paper calculates the local recovery error for |V| cutoff points iteratively and proposes to apply cutoff giving minimum local recovery error. Fig.3 shows the explanation of  $q_m$  and  $e_m$ .

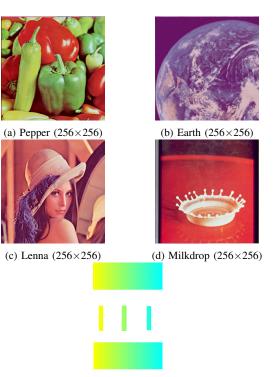
### IV. EXPERIMENTAL RESULTS

In this section, we conduct comparative experiments with Levin's method. Test images are shown in Fig. 4. In the proposed method, the number of divisions in superpixels is set to N=400 for all images, and  $\alpha = 0.1$  is used.

Table I shows the peak signal to noise ratio (PSNR) values and the number of given color pixels. Fig.5-9 show the given luminance images with several color pixels and the colorized results. The center pixels of each red circle in Fig.5-(b) to Fig.9-(b) have a given color pixel.

As can be seen, the proposed algorithm gives higher PSNR values than Levin's algorithm. Furthermore, from Fig.5 to 9, we can see that the results of Levin's algorithm have several faded colors. On the other hand, the proposed algorithm gives well colorized results.

Table II and III show the PSNR values of the colorized images when the 0.015 and 0.03 percent color information is given randomly, respectively. As can be seen, the proposed algorithm gives higher PSNR values than Levin's algorithm for all conditions. In particular, when less color information is given, the proposed algorithm achieves much higher colorization performance than Levin's algorithm. In the subjective comparison, the result of Levin has not been able to obtain a good image, the proposed method is totally colored and the color is properly applied to places not giving color.



(e) Gradation (300×464) Fig. 4: Test images

TABLE I: PSNR[dB] comparison for each image

Image	the number of the given color pic.	Levin	Our method
Pepper	4	18.3	21.4
Earth	4	24.5	30.1
Lenna	6	24.2	27.1
Milkdrop	5	19.0	21.4
Gradation	3	13.4	23.6

TABLE II: Comparison of PSNR[dB] at 0.015 percent(about 10 pixels)

Image	Levin's method	Our method
Pepper	17.32	18.89
Earth	25.78	30.01
Lenna	23.13	25.50
Milkdrop	18.85	20.67
Gradation	16.45	26.49

TABLE III: Comparison of PSNR[dB] at 0.03 percent(about 20 pixels)

Image	Levin's method	Our method
Pepper	18.65	20.99
Earth	26.65	31.13
Lenna	23.80	26.53
Milkdrop	20.42	22.66
Gradation	20.34	27.59

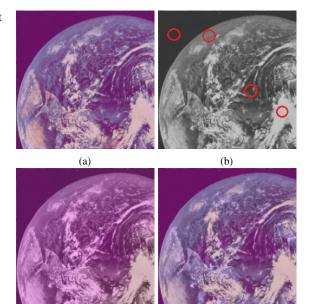


Fig. 6: (a)Orignal image (b)given image with 4 color pixels (c)Levin's method and (d)The proposed algorithm

(d)

(c)



Fig. 5: (a)Orignal image (b)given image with 4 color pixels (c)Levin's method and (d)The proposed algorithm

Fig. 7: (a)Orignal image (b)given image with 6 color pixels (c)Levin's method and (d)The proposed algorithm

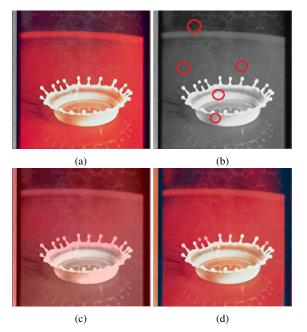


Fig. 8: (a)Orignal image (b)given image with 5 color pixels (c)Levin's method and (d)The proposed algorithm

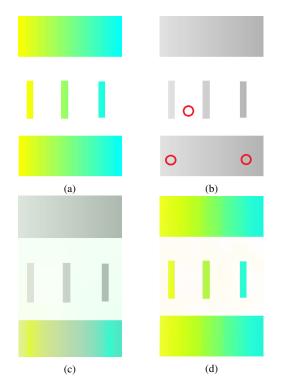


Fig. 9: (a)Orignal image (b)given image with 3 color pixels (c)Levin's method and (d)The proposed algorithm

## V. CONCLUSION

This paper proposes a method to obtain a natural color image even when a few color pixels are given. In order to achieve this, this paper introduces graph signal processing and superpixel based image segmentation technique. A graph is denoted on the given grayscale image, and important pixels selected by image segmentation are set as the vertices. The graph Fourier transform based signal recovery technique recovers a graph signal corresponding to the chrominance values of the important pixels. Because the connecting of the vertices affects the performance of the proposed algorthm, this paper proposes the connecting function based on the shpae of superpixels and luminance values. In the numerical experiments, the proposed method gives higher PSNR values than Levin's algorithm, and the proposed algorithm reduces the fading of color compared with the Levin's algorithm. From these results, even under conditions where there are few colors such that an image cannot be colorized by Levin's method, this paper showed that it can be colorized by the proposed method.

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