A Diversification Strategy for IIR Filter Design Using PSO

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***Abstract*—IIR (Infinite Impulse Response) filter design problem is a non-linear optimization problem. Because PSO (Particle Swarm Optimization) can enumerate solution candidates quickly, it is known as an effective method for such a problem. However, PSO has a drawback that tends to indicate a premature convergence due to a strong directivity. In this paper, PSS (Problem Space Stretch)-PSO is verified to avoid the local minimum stagnation. Several design examples are shown to present the effectiveness of the method.**

I. Introduction

IIR filter is a discrete time signal processing circuit used in various applications including a communication and a measurement. IIR filters can realize a sharp cutoff response with a lower filter order than FIR filters. However, IIR filters require a stability assurance and thus many local solutions exist in the design problem space.

Some methods were proposed for solving the such problem [1],[2]. In [1], SDP (Semidefinite programming) based method was used for solving the problem. In [2], Remez algorithm based method was used for the complex Chebyshev approximation design.

On the other hand, several heuristic methods were proposed [3]−[5]. Among them, PSO can enumerate solution candidates quickly. Therefore, PSO is effective for the IIR filter design problem where many local solutions exist in the problem space. However, PSO has a drawback that tends to indicate a premature convergence due to the strong directivity. Therefore, it is required to avoid the local minimum stagnation. Some methods were developed to avoid the local minimum stagnation [7]−[9].

In [7], the particle relocation method was proposed based on multiple swarms. When one of swarms stagnated, the relocation space is determined by the global bests of three swarms. The space determined from multiple local solutions may contain different local solutions from them. Then, all particles of the stagnation swarm are relocated within the convex hull spanned by the global bests of multiple swarms.

In [8], a penalty function is added to the objective function when the stagnation occurred. The objective function value of the local solution is temporally increased by the penalty function. As a result, each particle is prompted to escape from the stagnation point.

In [9], PSS-PSO stretches a problem space of the objective function when the stagnation occurred. Then, the gain of the desired response is changed. Although only the gain parameter of the frequency response is considered to be changed, PSO does not selectively update only one element of parameter vector. Therefore, the another parameters also change simultaneously and the local minimum stagnation can be avoided.

In this study, a search mechanism of PSS-PSO is verified from a point of view of the poles and the zeros of the designed filters. Several design examples are shown to present the effectiveness of the method.

II. Design problem

The frequency response of IIR filter is formulated in the following equation,

$$H(\omega) = a_0 \frac{\prod_{n=1}^{N}(1 - z_n e^{-j\omega})}{\prod_{m=1}^{M}(1 - p_m e^{-j\omega})} \tag{1}$$

where a_0 is a filter coefficient, z_n are zero points, p_m are poles, N is a numerator order and M is a denominator order. The design problem of IIR filter is expressed in a minimax criterion as following,

$$\min_{\boldsymbol{x}} \max_{\omega \in \Omega} |D(\omega) - H(\omega)|, \tag{2}$$

where $\boldsymbol{x} = [a_0, z_1, \cdots z_N, p_1, \cdots, p_M]^T$ is the design parameter vector, Ω is the approximate frequency band, ω is the normalized angular frequency, $D(\omega)$ is the desired response. The design problem is to determinate $\boldsymbol{x}$ so as to minimize the maximum error between $D(\omega)$ and $H(\omega)$ on the approximate frequency band.

III. IIR filter design using Particle Swarm Optimization

PSO is one of the multi-point search algorithms. PSO is consisted of multiple particles, each particle is specified by a location $\boldsymbol{x}$ and a speed $\boldsymbol{v}$. The updating procedure of the i-th particle is formulated as,

$$\boldsymbol{x}_i^{k+1} = \boldsymbol{x}_i^k + \boldsymbol{v}_i^{k+1}, \tag{3}$$

$$\boldsymbol{v}_i^{k+1} = w\boldsymbol{v}_i^k + c_1 r_1(\boldsymbol{pbest}_i^k - \boldsymbol{x}_i^k) + c_2 r_2(\boldsymbol{gbest}^k - \boldsymbol{x}_i^k), \tag{4}$$

where $\boldsymbol{pbest_i}$ is the best location of the i-th particle, $\boldsymbol{gbest}$ is the best location among all particle locations up to the k-th iterations, K is the number of iterations, r_1 and r_2 are

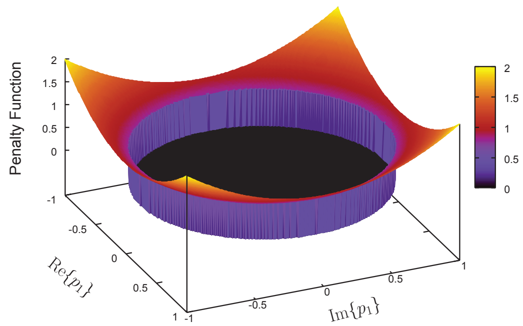

Fig. 1. Penalty Function (R=0.89)

uniform random numbers in the interval of [0, 1], w is the inertia weight parameter, c_1 is the weight parameter toward the $\boldsymbol{pbest_i}$ and c_2 is the weight parameter toward the $\boldsymbol{gbest}$. PSO has the strong directivity to the local solution. When PSO is applied to the IIR filter design problem, the objective function is defined as,

$$F(\boldsymbol{x}) = \max_{\omega\in\Omega} |D(\omega) - H(\omega)| + \varphi(p_{\max}), \quad (5)$$

where $\varphi(p_{\max})$ is a penalty function for assuring the stability [7].

The stability condition of the IIR filter is that all poles exist within the unit circle on z-plane. When using the minimax criterion, the error of the approximate band tends to concentrate in the transition band. Then, the magnitude ripple occurs in the transition band, and the poles tend to approach the unit circle. In $\varphi(p_{\max})$, a maximum pole radius is limited as following,

$$\varphi(p_{\max}) = \begin{cases} p_{\max}^2, & p_{\max} \geq R \\ 0, & \text{otherwise} \end{cases}, \quad (6)$$

$$p_{\max} = \max_{m=1,\cdots,M} |p_m|, \quad (7)$$

where R is the prescribed maximum pole radius. The penalty function of $R = 0.89$ is shown in the Fig.1.

IV. IIR filter design using PSS-PSO

PSS-PSO [9] was proposed to avoid the local minimum stagnation. PSS-PSO stretches the problem space of the objective function when the stagnate occurred. As a result, the objective function value of the stagnation point is changed, the local minimum stagnation can be avoided. When PSS-PSO is applied to the IIR filter design problem, the objective function is defined as,

$$F(\lambda, \boldsymbol{x}) = \max_{\omega\in\Omega} |\lambda D(\omega) - H(\omega)| + \varphi(p_{\max}), \quad (8)$$

where λ is an uniform random numbers in the interval of $[1-\gamma, 1+\gamma]$, γ is a scalar. If $\lambda < 1$, the problem space is stretched, and if $\lambda > 1$, it is shrunk. When the gain of $D(\omega)$ is multiplied by λ, it can be considered that just a_0 of $H(\omega)$ is multiplied by λ. Because PSO does not selectively update only one element of $\boldsymbol{x}$, z_n and p_m also change simultaneously. As a result, diversification is promoted when the search is repeated.

TABLE I
Design conditions

	P	N	M	R	τ	f_p	f_s
Ex.1	80	6	4	0.88	4.0	0.20	0.30
Ex.2	90	8	6	0.89	6.0	0.20	0.27
Ex.3	140	12	8	0.93	10.0	0.20	0.24

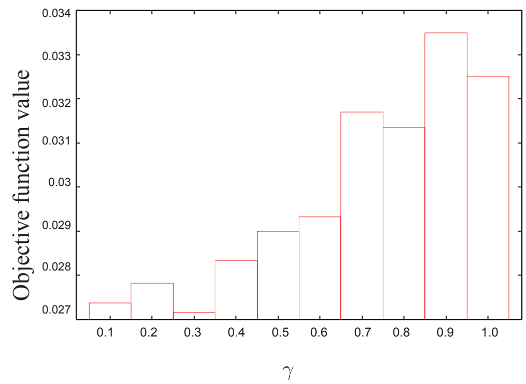

Fig. 2. Verification of γ of Ex.1

V. Verification

Design performance of PSS-PSO was verified in [9]. However, a search mechanism of PSS-PSO is unknown. For the verification of search mechanism, the variation of design parameters between before and after of the multiplication of λ to $D(\omega)$, and then the variation of the value of objective functions were investigated.

Three design examples are attempted for the verification. The desired frequency response is defined as,

$$D(\omega) = \begin{cases} e^{-j\omega\tau}, & 0 \leq \omega \leq 2\pi f_p \\ 0, & 2\pi f_s \leq \omega \leq \pi \end{cases}, \quad (9)$$

where P is a number of particle, τ is a group delay, f_p is a pass band edge frequency, f_s is a stop band edge frequency. Design conditions are shown Table I.

A. Parameters of PSO

The number of iterations was set to $K = 50,000$, the number of trials was set to $T = 50$, and number of frequency divisions was set to $S = 300$. Initial value of a_0 was set to $[-0.5, 0.5]$, z_n were set to $[-1.5, 1.5]$ and p_m were set to $[-R, R]$, $w = 0.5$, $c_1 = 1.0$, $c_2 = 2.6$. In general, PSO sets c_1 and c_2 to similar values. As a result, the diversification and the intensification of search are balanced. When c_2 is set to be larger than c_1, the intensification ability becomes stronger and the frequencies of the stagnations increase. Because many local solutions are enumerated by the stagnation avoidance, many local solutions are also enumerated because PSS-PSO can avoid stagnation.

γ was verified under design conditions in Table I. The objective function values for γ are shown from Fig.2 to Fig.4. From those results, we set γ to 0.2.

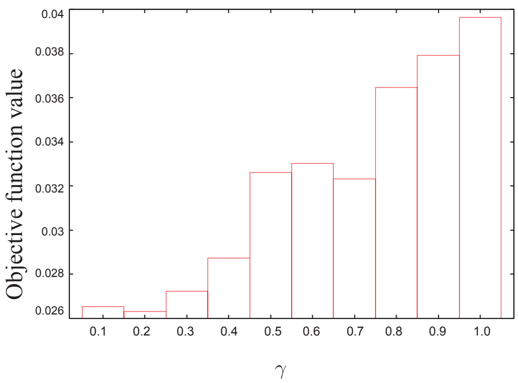

Fig. 3. Verification of γ of Ex.2

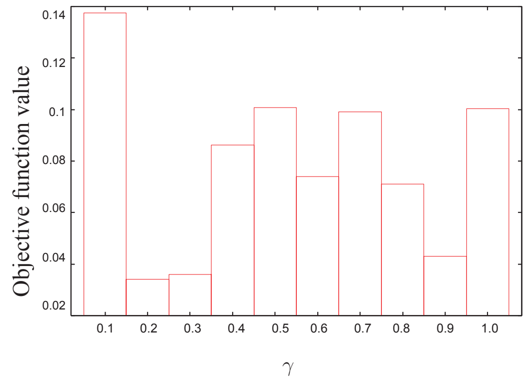

Fig. 4. Verification of γ of Ex.3

B. Verification result

The verification results are shown from Table II to Table VII. H indicates that the element of parameter is changed higher than the average value. L indicates that the element of parameter is changed lower than the average value. $Improvement$ is the number of improvements of the objective function value. $Worse$ is the number of worses of the objective function value. Sum is the total number of $Improvement$ and $Worse$. It is assumed to be a global search when the z_n or the p_m are H. It is assumed to be a local search when the z_n and the p_m are L.

From the results of Table II, Table IV and Table VI, in the case of HHH, HHL, it can be confirmed that the number of worses tends to be larger than the number of improvements. These results show that the global search found for solutions having different design parameters before and after the stagnation avoidance. On the other hand, it can be confirmed that the number of improvements are larger than the number of worses in the local search, i.e., the search of LLL. In addition, the frequencies of local searches is extremely large. From the results of Table III, Table V and Table VII, it can be confirmed that the best solution is solution of improvements in the local search. This fact reveals that a coarse-to-fine search is carried out stochastically in PSS-PSO.

The allocation of poles of the best solution are shown from Fig.5 to Fig.7. Those allocations reveal that the stability is assured in the filters designed. The magnitude response are shown from Fig.8 to Fig.10. From those results, it can be confirmed that the excessive ripple was suppressed in the transition band. The updating curves are shown from Fig.11 to Fig.13. When the stagnation occurred, the values of objective function change temporally because of the problem space stretch. That is, large variations in those curves correspond to the occurrence of the stagnation avoidance. Thus, the effectiveness of PSS-PSO is also confirmed even in the updating curves.

TABLE II
VERIFICATION RESULTS (EX.1)

a_0	z_n	p_m	$Improvement$	$Worse$	Sum
H	H	H	1522	1713	3235
H	H	L	1255	1477	2732
H	L	H	590	470	1060
H	L	L	904	835	1739
L	H	H	527	349	876
L	H	L	906	671	1577
L	L	H	1958	1132	3090
L	L	L	9488	5885	15373

VI. CONCLUSIONS

In this paper, the search mechanism of PSS-PSO was verified from the viewpoint of design parameter vector variation. From verification results, it was shown that PSS-PSO achieves the good design by the stochastic coarse-to-fine search.

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TABLE III
THE BEST SOLUTION OF OBJECTIVE FUNCTION VALUE FOUND FOR EACH COMBINATION (EX.1)

a_0	z_n	p_m	$Improvement$	$Worse$
H	H	H	0.026302	0.026718
H	H	L	0.026316	0.026766
H	L	H	0.026585	0.027136
H	L	L	0.026354	0.026840
L	H	H	0.026328	0.026411
L	H	L	0.026270	0.026370
L	L	H	0.026339	0.026451
L	L	L	0.026264	0.026292

TABLE IV
VERIFICATION RESULTS (EX.2)

a_0	z_n	p_m	$Improvement$	$Worse$	Sum
H	H	H	1563	2044	3607
H	H	L	1374	1912	3286
H	L	H	667	677	1344
H	L	L	1308	1299	2607
L	H	H	756	556	1312
L	H	L	1279	1045	2324
L	L	H	1866	976	2842
L	L	L	10191	5157	15348

TABLE V
THE BEST SOLUTION OF OBJECTIVE FUNCTION VALUE FOUND FOR EACH COMBINATION (EX.2)

a_0	z_n	p_m	$Improvement$	$Worse$
H	H	H	0.023726	0.024438
H	H	L	0.022791	0.023958
H	L	H	0.024316	0.024637
H	L	L	0.022487	0.024067
L	H	H	0.023952	0.024403
L	H	L	0.022591	0.022570
L	L	H	0.022678	0.023273
L	L	L	0.022444	0.023021

TABLE VI
VERIFICATION RESULTS (EX.3)

a_0	z_n	p_m	$Improvement$	$Worse$	Sum
H	H	H	1487	2040	3527
H	H	L	1591	2153	3744
H	L	H	961	913	1874
H	L	L	1573	1643	3216
L	H	H	891	695	1586
L	H	L	1715	1613	3328
L	L	H	2206	1267	3473
L	L	L	11063	5514	16577

TABLE VII
THE BEST SOLUTION OF OBJECTIVE FUNCTION VALUE FOUND FOR EACH COMBINATION(EX.3)

a_0	z_n	p_m	$Improvement$	$Worse$
H	H	H	0.023914	0.025280
H	H	L	0.024593	0.025461
H	L	H	0.023921	0.024119
H	L	L	0.022904	0.024068
L	H	H	0.024876	0.026397
L	H	L	0.023516	0.026758
L	L	H	0.023675	0.024622
L	L	L	0.022752	0.023059

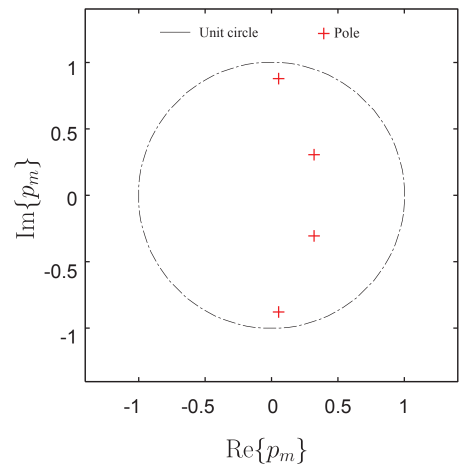

Fig. 5. Pole allocation of Ex.1

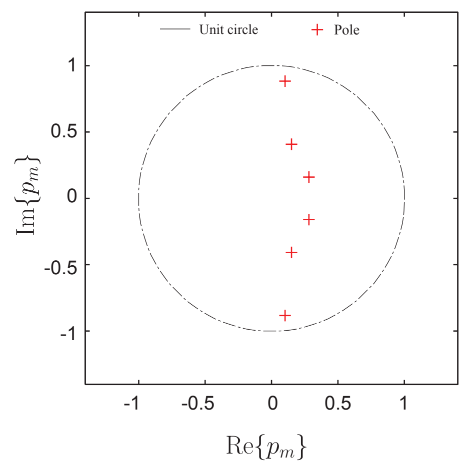

Fig. 6. Pole allocation of Ex.2

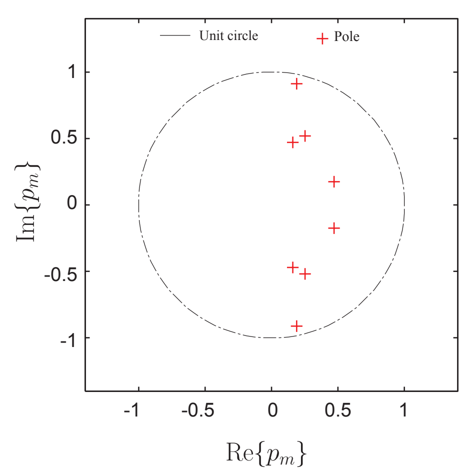

Fig. 7. Pole allocation of Ex.3

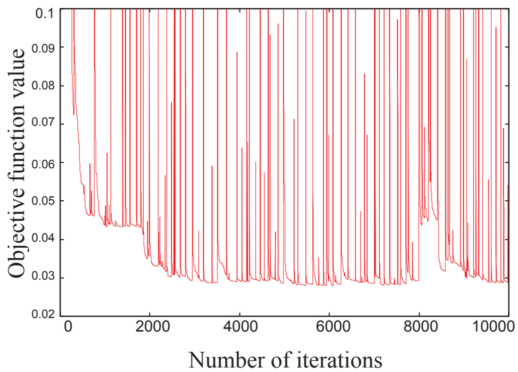

Fig. 8. Updating Curve of Ex.1

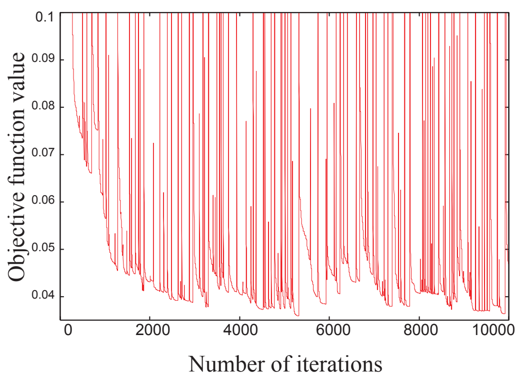

Fig. 9. Updating Curve of Ex.2

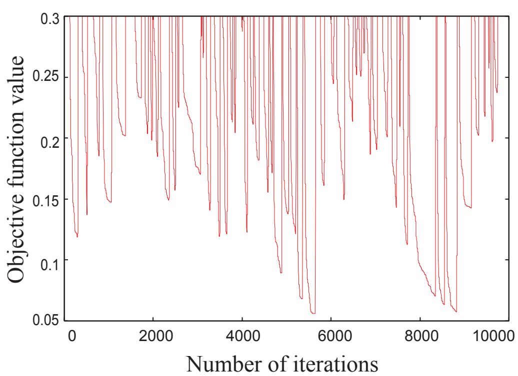

Fig. 10. Updating Curve of Ex.3

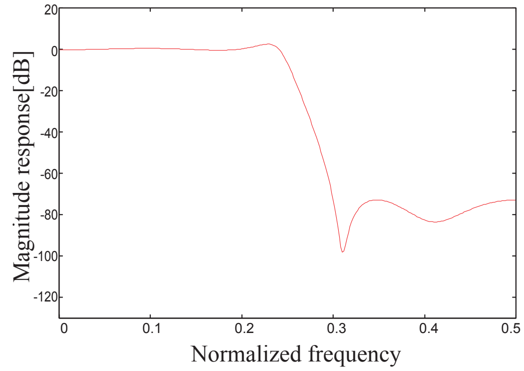

Fig. 11. Magnitude response of Ex.1

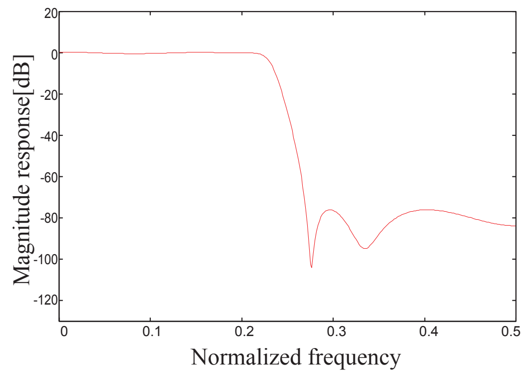

Fig. 12. Magnitude response of Ex.2

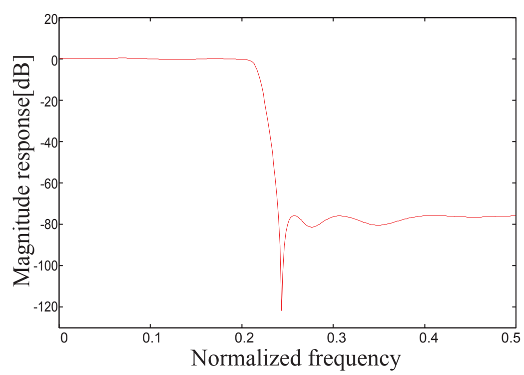

Fig. 13. Magnitude response of Ex.3