Design of Gradient and Variable Step-Size for Fast Adaptive Notch Filter

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***Abstract*—This paper proposes a new adaptive algorithm for gradient based adaptive notch filter in order to improve the convergence speed. We develop a new gradient by combining the gradients of two notch filters with the different rejection bandwidth. We also develop a variable step-size on the basis of the difference of the impulse response between two notch filters. Through the experiment, we reveal the effectiveness of the proposed method.**

I. Introduction

Narrow-band noise suppression has an important role in many applications of speech processing such as speech enhancement [1], [2], acoustic feedback control [3], a hum noise reduction in electrocardiograms [4], [5], and so on. Most of such applications require the fast and accurate noise reduction performance.

One of the typical methods for narrow-band noise suppression is an adaptive notch filter [6]. The notch filter is composed of a second-oder infinite impulse response (IIR) filter which has one null frequency called a notch frequency. The notch filter can suppress the narrow-band noise when the notch frequency is equal to the noise frequency. The adaptive notch filter is useful for automatically estimating the noise frequency. The adaptive notch filter adjusts the notch frequency so as to minimize the error function, which is defined as the mean square error of the filter output. Such an adaptive algorithm is called stochastic gradient descent (SGD) method. The convergence performance of SGD depends on the magnitude of the gradient and the length of the impulse response.

Among so many kinds of the SGD algorithm, the normalized least mean square (NLMS) algorithm [7] is generally used for updating the notch frequency because of its simplicity. The gradient of NLMS varies in accordance with the distance between the notch frequency and the noise frequency. Specifically, the gradient becomes large as the notch frequency is close to the noise frequency. This means that NLMS algorithm provides a fast convergence along with decreasing the frequency error, but the convergence speed is too slow when the notch frequency is far from the noise frequency. This problem is remarkable when the rejection bandwidth of the notch filter is narrow.

In order to improve the convergence speed, many researchers have developed various types of the adaptive notch filters [10]-[14]. Most of them are based on the multiple notch filters. Piloted notch filter [10], [11] and parallel composite notch filter [12], [13] are the particularly superior methods among them. They modify the gradient characteristics where the notch frequency is far from the noise frequency. However, they have not been a fundamental solution because they do not consider the effect of the impulse response.

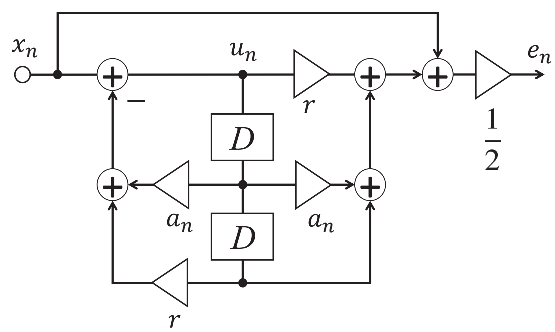

Fig. 1. Structure of the notch filter.

In this paper, we proposes a new adaptive algorithm to achieve fast convergence. The proposed gradient is designed by combining two notch filters with the different rejection bandwidth. Additionally, we introduce a new variable step-size using the difference of the impulse response of two notch filters. Through the experiments, we show the effectiveness of the proposed method.

II. Adaptive Notch Filter

In this section we show a structure of the notch filter and a behavior of the NLMS algorithm in the adaptive notch filter.

A. Structure

A second-order adaptive IIR notch filter [6] is widely used for estimating a narrow-band noise. Figure 1 illustrates a structure of the IIR notch filter. Here, x_n is the input signal, e_n is the output signal, and u_n is the signal generated by $u_n = x_n - au_{n-1} - ru_{n-2}$, where the parameter a_n determines a null frequency called notch frequency and the parameter r $(-1 < r < 1)$ controls the rejection bandwidth. In this paper the input signal is assumed to consist of a sinusoidal noise

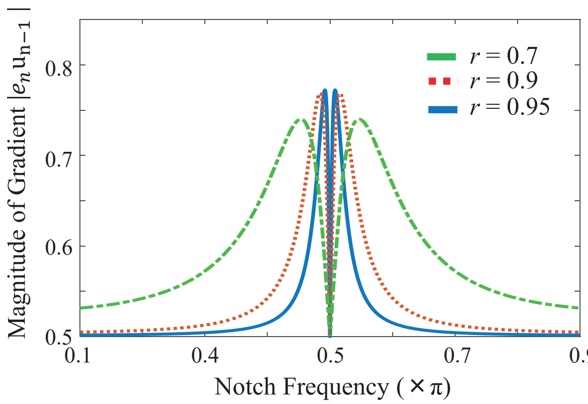

Fig. 2. Magnitude of the gradient.

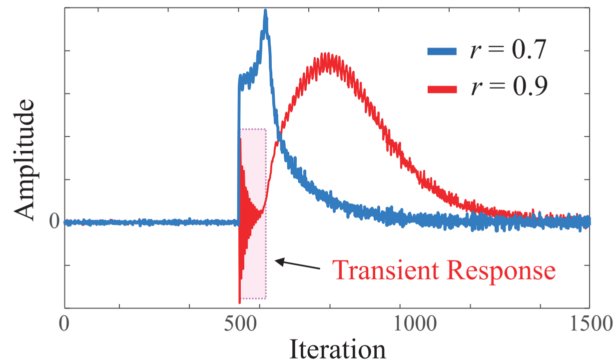

Fig. 3. Behaviors of the gradient for NLMS algorithm with varying r when the sinusoidal input signal is changed at 500 iterations.

and wide-band desired signal. Letting the sinusoidal noise and wide-band signal be denoted by s_n and w_n, respectively, the input signal x_n is described by

$$x_n = w_n + s_n, \tag{1}$$
$$s_n = p\cos(\omega_s n + \phi), \tag{2}$$

where ω_S is the noise frequency, p is the amplitude of noise, and ϕ is the phase of noise.

The transfer function of the notch filter is

$$H(z) = \frac{1}{2}\left(1 + \frac{r + az^{-1} + z^{-2}}{1 + az^{-1} + rz^{-2}}\right). \tag{3}$$

The parameter a is related to the notch frequency ω_N as follows:

$$a = -(1 + r)\cos(\omega_N). \tag{4}$$

The adaptive notch filter can estimate and remove the narrow-band noise by adjusting a so that $\omega_N = \omega_s$.

B. Normalized LMS algorithm

The Normalized LMS (NLMS) algorithm [7] is a stochastic gradient algorithm generally used for updating a. In this algorithm, the updating term is derived from the gradient of the error function $E[|e_n|^2]$. The updated value of a is calculated by

$$a_{n+1} = a_n - \mu\frac{e_n u_{n-1}}{E[u_{n-1}^2]}, \tag{5}$$

where μ is a step-size parameter which adjusts the convergence speed and amount of the misadjustment defined by the estimation error in the steady state. In (5), the gradient term, $e_n u_{n-1}$, is normalized by the tap-input power $E[u_{n-1}^2]$.

The convergence speed of the NLMS depends on not only the gradient but also the impulse response of the notch filter. The magnitude of the gradient varies in accordance with the frequency estimation error given by $|\omega_N - \omega_S|$. Figure 2 plots theoretical curves of the magnitude of the gradient versus the notch frequency ω_N under varying r as $0.7, 0.9, 0.95$. The noise frequency ω_S is set to $\omega_S = \pi/2$. As seen in this figure, the magnitude of gradient tends to be relatively low when ω_N is far from ω_S. This fact implies that the convergence speed slows down when $|\omega_N - \omega_S|$ is large. The magnitude of the gradient can be raised by using large bandwidth parameter r when $|\omega_N - \omega_S|$ is large, while the magnitude of the gradient decreases conversely in the vicinity of ω_S.

The long impulse response also affects the convergence speed. Here we assume that the length of the impulse response is measured by the time constant, even though a notch filter has an infinite impulse response actually. Namely, the impulse response is assumed to be long when the time constant is large. In this case, the adaptive notch filter with large r provides slow convergence because of the large time constant. To confirm the effect of the impulse response, we carried out the experiment of the sinusoidal noise estimation. For this experiment, we set the noise frequency to $\omega_S = 0.5\pi$ at $0 \sim 499$ iterations and $\omega_S = 0.1\pi$ at $500 \sim 2500$ iterations. The sinusoidal noise is added to the desired white Gaussian signal so that SNR is 10 dB.

Figure 3 shows two curves of the gradient versus the number of iterations, which are corresponding to $r = 0.7, 0.9$, respectively. Here, the notch frequency is updated by using (5). As seen in Fig. 3, the adaptive notch filter with $r = 0.9$ needs the long transient response time illustrated by the pink region to obtain the reliable gradient value. Conversely, in the case of the smaller r that is $r = 0.7$, the amplitude of the gradient increases rapidly at 500 where the noise frequency is changed, because of the short impulse response. After that, however, the amplitude of the gradient also decreases rapidly with decreasing the estimation error. This result indicates that there is a trade-off relation between the convergence speed far from ω_S and the one in the vicinity of ω_S.

III. Proposed Gradient Function

In order to overcome the trade-off restriction in the NLMS algorithm, we develop a new adaptive algorithm by combining two notch filters with varying r. The structure for the new algorithm is depicted in Figure 4. In this structure, two notch

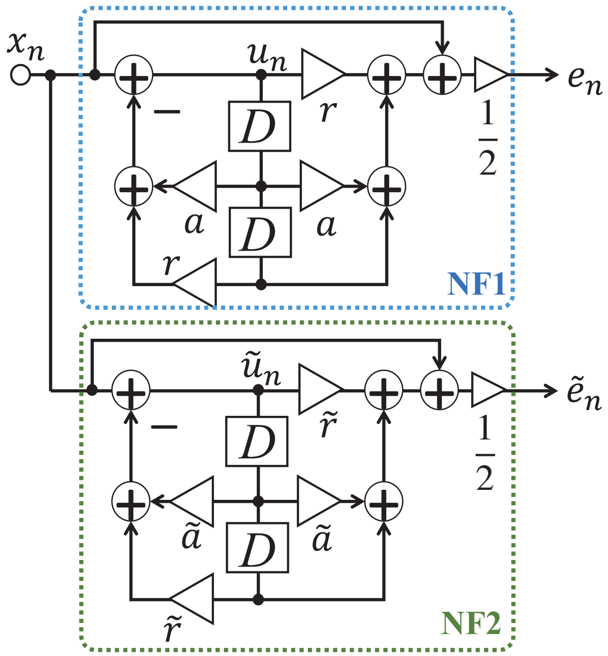

Fig. 4. Structure of the notch filter for the proposed algorithm

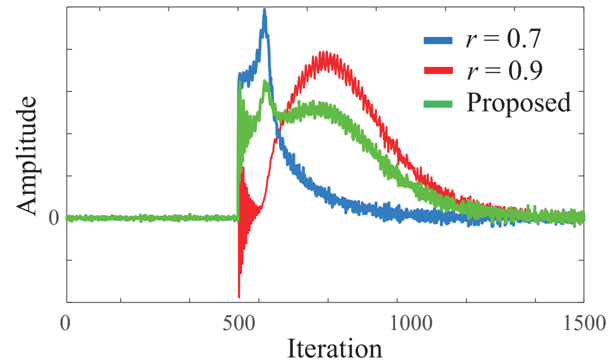

Fig. 5. Behaviors of the gradient for the proposed algorithm and NLMS algorithm with varying r when the sinusoidal input signal is changed at 500th iteration.

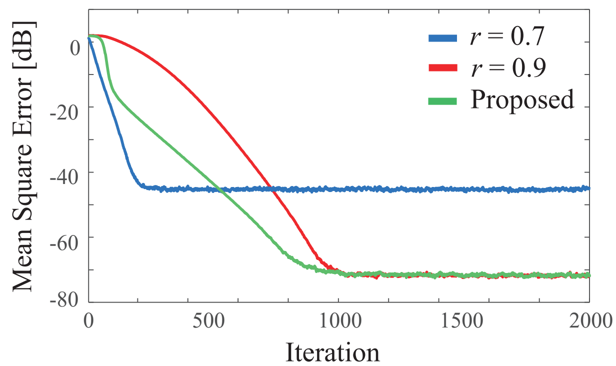

Fig. 6. MSE curves for the proposed algorithm and NLMS algorithm with varying r as $r = 0.7, 0.9$.

filters are connected in parallel. The upper notch filter (NF1) has a large bandwidth parameter r. The lower notch filter (NF2), which is used only for calculating the gradient, has a smaller bandwidth parameter $\tilde{r}$ than the upper one. In the proposed algorithm, the gradient is defined by summation of two gradients obtained from NF1 and NF2. Namely, the notch frequency parameter of NF1, a_n, is updated by

$$a_{n+1} = a_n - \mu \frac{(1-r)e_n u_{n-1} + (1-\tilde{r})\tilde{e}_n \tilde{u}_{n-1}}{E[u_{n-1}^2]}. \tag{6}$$

In the second term on the light side of (6), two gradients for NF1 and NF2 are weighted with $(1 - r)$ and $(1 - \tilde{r})$, respectively, in order to equalize the amount of mutual misadjustment [6].

Both the notch frequency parameters a_n and $\tilde{a}_n$ should be set to have a common notch frequency. Using the updated a_n, the updation of $\tilde{a}_n$ is done easily by

$$\tilde{a}_n = \frac{(1 + \tilde{r})}{(1 + r)} a_n. \tag{7}$$

To confirm that the proposed algorithm can break through the trade-off restriction, we carried out an experiments. The settings are the same as the above experiment in Sec. II. Figure 5 plots the behaviors of the gradient for the proposed algorithm, NLMS algorithm with $r = 0.7$, and NLMS algorithm with $r = 0.9$ versus the number of iterations. As seen in this figure, the gradient in the proposed algorithm can keep a high magnitude until the start of the steady-state. Figure 6 shows the MSE curves for the three algorithms. Here the noise frequency is fixed to $\omega_S = 0.1\pi$ while the initial notch frequency is set to 0.5π. MSE is evaluated by

$$\text{MSE} = 10 \log_{10} |\omega_N - \omega_S| \quad [\text{dB}]. \tag{8}$$

It can be seen from Fig. 6 that the proposed algorithm improves the convergence speed at the beginning of adaptation, compared with NLMS algorithm with $r = 0.9$. However, there still remains a problem that the convergence time is not improved even by the proposed algorithm. Next, we investigate a variable step-size.

A. Variable Step-size

To further improve the convergence speed, we develop a variable step-size for the proposed algorithm. The variable step-size should be changed so as to be large with increasing the estimation error. For evaluating the estimation error, we focus on the difference of the output residual power between NF1 and NF2. The output residual signal is defined by

$$r_n = x_n - e_n. \tag{9}$$

The power of r_n is equal to the noise power p^2 approximately only when $|\omega_N - \omega_S| = 0$ and also decreases with increasing

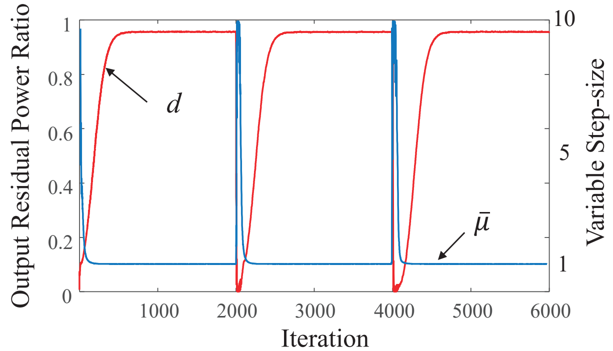

Fig. 7. Behaviors of the output residual power ratio d and the variable step-size $\bar{\mu}$.

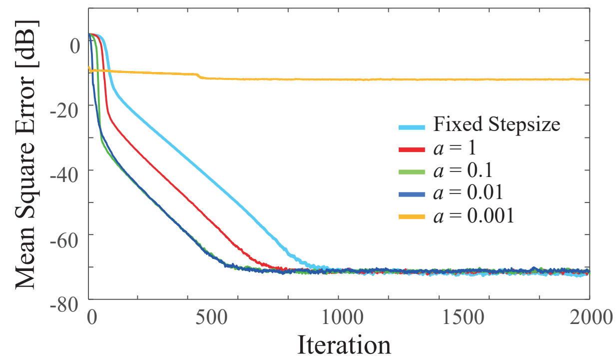

Fig. 8. Behaviors of the estimated noise frequency for the proposed algorithm with the variable step-size under varying α as $\alpha = 1, 0.1, 0.01$ and 0.001.

the estimation error. Now we introduce a power ratio function defined by

$$\begin{aligned} d_n &= \frac{E[r_n^2]}{E[\tilde{r}_n^2]} \\ &= \frac{E[(x_n - e_n)^2]}{E[(x_n - \tilde{e}_n)^2]} \\ &\approx \frac{E[x_n^2] - E[e_n^2]}{E[x_n^2] - E[\tilde{e}_n^2]}, \end{aligned} \tag{10}$$

where

$$\tilde{r}_n = x_n - \tilde{e}_n. \tag{11}$$

The function d_n takes 1 when both e_n and $\tilde{e}_n$ are in the steady state, whereas d is close to 0 when e_n or $\tilde{e}_n$ is in the transient. This is because the relation of $E[e_n^2] \ll E[\tilde{e}_n^2]$ holds only when the noise frequency is changed, since NF1 has the longer impulse response than NF2. Using the characteristics of d_n, the proposed variable step-size is derived by

$$\bar{\mu} = \frac{1+\alpha}{d_n + \alpha}\mu, \tag{12}$$

where α is a small constant.

We confirm the behavior of the variable step-size through an experiment. The settings in this experiment are the same as the previous experiments except for the noise frequency. The noise frequency follows

$$\omega_S = \begin{cases} 0.75\pi, & 1 \le n < 2000 \\ 0.25\pi, & 2000 \le n < 4000 \\ 0.5\pi, & 4000 < n \le 6000 \end{cases}. \tag{13}$$

Figure 7 plots the behaviors of d and $\bar{\mu}$ with $\alpha = 0.01$ versus the number of iterations. The output residual power ratio d is close to 1 in the steady state, and is almost 0 when the noise frequency is changed. In the transient state, d increases from 0 toward to 1 with decreasing the estimation error. The variable step-size suddenly increases when the noise frequency is changed, and also converges to 1 in the steady state. Figure 8 shows the behaviors of the estimated noise frequency for the proposed algorithm with varying α. The parameter α is varied to $1, 0.1, 0.01$ and 0.001. For comparison, the result of the proposed algorithm with fixed step-size is also plotted. We plot the results in only the range of $0 \sim 2000$ iterations. It is obvious that the convergence time becomes short with decreasing α, except in the case of $\alpha = 0.001$. In the following experiments we set α to 0.01.

IV. Experiments

To reveal the effectiveness of the proposed method, we carried out experiments for narrow-band noise reduction. For comparison, we used the following 4 conventional algorithms: NLMS algorithm [7], Piloted notch filter [10], Parallel composition notch filter [12], MIG algorithm [14].

The input signal consists of a white Gaussian desired signal and a single sinusoidal noise whose SNR is set to 10dB. The noise frequency follows (13). The other settings are as followings: the bandwidth parameter of NF1 is $r = 0.9$, the bandwidth parameter of NF2 is $\tilde{r} = 0.7$, the variable step-size parameter is $\alpha = 0.01$, and the fixed step-size parameter $\mu = 0.01$. The step-size parameters of the conventional algorithms are adjusted so that each amount of misadjustment is equivalent to the one of the proposed algorithm. Figure 9 shows the behaviors of the notch frequency for 5 methods. As seen in this figure, the proposed algorithm can track the noise frequency fastest in all methods. Figure 10 shows the MSE curves for 5 methods. We can also confirm from this figure that the proposed method always provides shortest convergence time. Figure 11 is an enlarged figure of Fig. 10 in the range of $2000 \sim 4000$. The proposed algorithm converges to the steady state by 500 iterations. This is about two times as short as the conventional NLMS algorithm.

V. Conclusion

In this paper, we proposed a new gradient and a new variable step-size of the adaptive notch filter for fast convergence. The proposed algorithm provides two times as short convergence time as the conventional NLMS method.

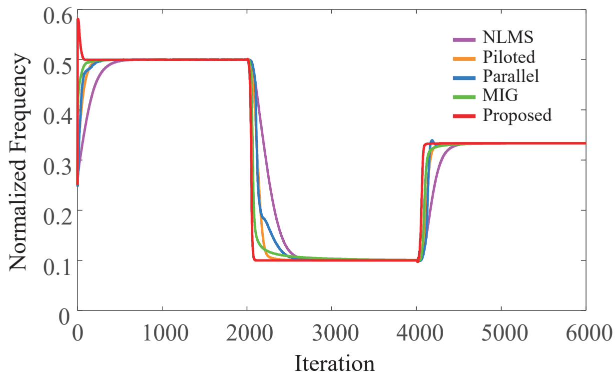

Fig. 9. Behaviors of the notch frequency for the proposed algorithm and 4 conventional algorithms.

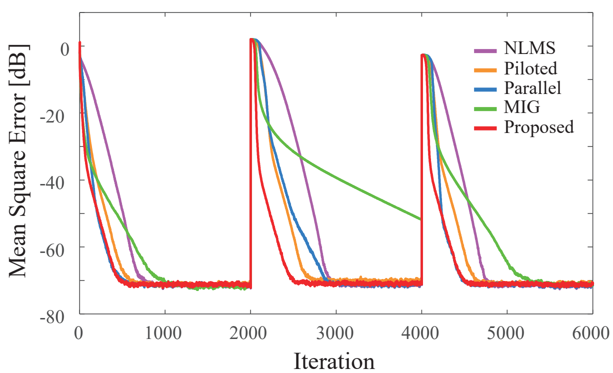

Fig. 10. MSE curves for the proposed algorithm and 4 conventional algorithms.

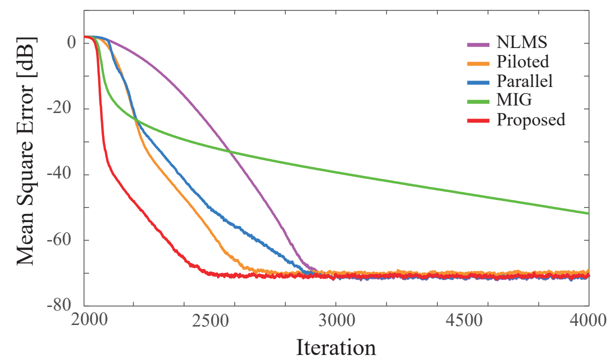

Fig. 11. MSE curves for the proposed algorithm and 4 conventional algorithms in the range of 2000 $\sim$ 4000.

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