Adaptive Matching Pursuit Method Based on Auxiliary Residual for Sparse Signal Recovery

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***Abstract*—Greedy pursuit methods are widely used for compressive sensing (CS) and sparse signal recovery due to their low computational complexity. In this paper an adaptive matching pursuit is proposed, which is based on the backtracking-based adaptive orthogonal matching pursuit (BAOMP) and uses auxiliary residual to make correlation test to add more correct atoms per iteration. The proposed method can be regarded as an improved BAOMP. The simulation results show that it has better performance to those of some other greedy pursuit methods. Finally the experiment of CS-based ISAR imaging verifies the effectiveness of the proposed method.**

I. Introduction

Compressive sensing (CS) theory is a novel data sampling and compression approach, which can recover sparse or compressive signals with lower rates than their Nyquist rates [1-3]. Consider a K -sparse signal $\mathbf{x} \in R^N$ that has at most K ($K \ll N$) nonzero entries. $S = \{i | x_i \neq 0, i = 1, 2, \cdots, N\}$ denotes the support of $\mathbf{x}$. Using a $M \times N$ ($M < N$) measurement matrix $\mathbf{\Phi}$, we can get a $M \times 1$ compressed measurement vector

$$\mathbf{y} = \mathbf{\Phi x} \tag{1}$$

where $\mathbf{\Phi} = [\phi_1, \phi_2, \cdots, \phi_N]$ and the column ϕ_i is also called an atom. Since $M < N$, the reconstruction of $\mathbf{x}$ from $\mathbf{y}$ is generally ill-posed. According to CS theory, if $\mathbf{\Phi}$ satisfies the restricted isometry property (RIP), then the K -sparse signal $\mathbf{x}$ can be reconstructed from only $M = O(K \log N)$ measurements through some non-linear recovery methods [2].

Among the existing recovery methods, greedy pursuit algorithms [4-15] have received significant attention due to their low complexity and competitive reconstruction performances. The main idea of greedy algorithms is to estimate the support of the sparse signal iteratively and construct an approximation on the estimated support until a certain stopping condition is satisfied. The classical greedy algorithms include orthogonal matching pursuit (OMP) [4], stagewise OMP (StOMP) [5], regularized OMP (ROMP) [6], generalized OMP (GOMP) [7], compressive sampling matching pursuit (CoSaMP) [8], subspace pursuit (SP) [9], sparsity adaptive matching pursuit (SAMP) [10], backtracking-based adaptive OMP (BAOMP) [11], and randomly enhanced adaptive SP (REASP) [12] etc.. Recently some improved greedy algorithms have also been proposed in [13-15]. The adaptive reduced-set matching pursuit (ARMP) algorithm in [13] adaptively chose some atoms by setting appropriate thresholds. Maximum a posterior (MAP) support detection [14] and global optimization searching technique [15] are respectively used in the greedy algorithm to estimate the support of the sparse signal.

In the above greedy algorithms, backtracking-based greedy algorithms are more attractive, which can remove unreliable atoms after adding several correlative atoms into the estimated support set per iteration. Backtracking-based greedy algorithms have superior performance. According to the atoms addition and deletion techniques, the backtracking-based greedy algorithms can be divided into two types. The first one takes fixed number of atoms, such as CoSaMP and SP. If the sparsity K is known a prior, SP adds K atoms into the estimated support and then deletes K atoms from the estimated support [9]. The other one uses adaptive techniques to add and delete atoms with preset parameter thresholds, such as BAOMP [11] and ARMP[13]. The ARMP algorithm needs the information of sparsity and the BAOMP does not require such a priori knowledge. It has shown that the BAOMP is better than those of some other greedy algorithms.

Combining different greedy pursuit algorithms can improve the recovery performance. In this paper, based on the adaptive technique in BAOMP and fixed scale technique in SP, we propose a modified BAOMP called auxiliary residual based adaptive matching pursuit (AR-AMP). The main idea of AR-AMP is to compute auxiliary residual to make correlation test to add more correct atoms per iteration. The above two different atoms' addition and deletion techniques are used in each iteration. The AR-AMP can be treated as a generalization of BAOMP. The experimental results demonstrate that the AR-AMP is superior to some other greedy pursuit methods. Finally we apply it to CS-based ISAR imaging to verify its reconstruction performance.

II. Algorithm Description

In this section, we will introduce the BAOMP briefly and then the proposed AR-AMP will be presented in detail.

A. *Review of the BAOMP*

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Firstly, some notations used in this paper are given below. $\mathbf{\Phi}_\Lambda$ represents a sub-matrix of $\mathbf{\Phi}$ with columns indexed by Λ. Similarly $\mathbf{x}_\Lambda$ is the sub-vector of $\mathbf{x}$ containing the entries indexed by Λ. Superscripts $*$ and † refer to the transpose and Moore-Penrose pseudo inverse of a matrix. $|\cdot|$ and $\langle \cdot,\cdot \rangle$ denote the absolute value and inner product, respectively. $card(\cdot)$ computes the cardinality of a set.

The BAOMP algorithm use preset parameter thresholds μ_1 and μ_2 to add and delete atoms, its each iteration (e.g., the k-th iteration) is composed of three major steps.

1) Atoms' addition: choose several atoms that have the largest absolute correlation with the residual $\mathbf{r}^{k-1}$, i.e., compute the candidate set C^k, whose element i satisfies: $\left|\langle \mathbf{r}^{k-1}, \varphi_i \rangle\right| \ge \mu_1 \cdot \max_{j\in\{1,2,\ldots,N\}} \left|\langle \mathbf{r}^{k-1}, \varphi_j \rangle\right|$, and then add them into the estimated support set to obtain $T^{k-1} \cup C^k$.

2) Atoms' deletion: compute $\mathbf{x}^k_{T^{k-1}\cup C^k} = \mathbf{\Phi}^\dagger_{T^{k-1}\cup C^k}\mathbf{y}$ and find some entries in $\mathbf{x}^k_{T^{k-1}\cup C^k}$ which satisfies $\left|x^k_b\right| < \mu_2 \cdot \max\left|\mathbf{x}^k_{C^k}\right|$, then delete the corresponding indices from $T^{k-1} \cup C^k$ and obtain the estimated support set T^k.

3) Update estimate and residual: calculate the new solution $\mathbf{x}^k_{T^k}$ as well as its residual $\mathbf{r}^k = \mathbf{y} - \mathbf{\Phi}_{T^k}\mathbf{x}^k_{T^k}$.

In the BAOMP algorithm, $\mathbf{x}^k_{C^k}$ represents the sub-vector of $\mathbf{x}^k_{T^{k-1}\cup C^k}$ containing the entries indexed by C^k. Step 2 uses the backtracking strategy to discard the atoms whose coefficients are smaller than that of the currently chosen atoms.

B. The proposed AR-AMP

From the estimated support set T^k in the BAOMP, the atoms having the largest elements of $\mathbf{x}^k_{T^k}$ may be correct with high probability, thus we can choose them and construct a subset of T^k, called auxiliary estimated support set Γ^k, then compute the corresponding auxiliary residual to make correlation test, therefore we can obtain a complementary candidate set $\boldsymbol{C}^k_a$, which can include some penitential correct atoms. Thus we propose an auxiliary residual based adaptive matching pursuit (AR-AMP), which can produce two correlative sets and add them into the estimated support set to identify more potential correct atoms. The flow chart of the propose AR-AMP algorithm is shown in Fig.1. The detailed description of its procedure is given below.

Auxiliary residual based adaptive matching pursuit (AR-AMP)

Input: measurement matrix $\mathbf{\Phi}$, measurement vector $\mathbf{y}$, atom-adding constant threshold $\mu_1 \in [0,1]$, atom-deleting constant threshold $\mu_2 \in [0,1]$, halting threshold ε, number of maximum iterations $k_{\max}$.

Initialization: estimated signal $\mathbf{x}^0 = \mathbf{0}$, residue $\mathbf{r}^0 = \mathbf{y}$, estimated support set $T^0 = \varnothing$, iteration number $k = 1$, auxiliary residue $\mathbf{r}^0_a = \mathbf{r}^0$.

Iteration:

1 Atoms' selection: compute the candidate set C^k, whose element i satisfies: $\left|\langle \mathbf{r}^{k-1}, \varphi_i \rangle\right| \ge \mu_1 \cdot \max_{j\in\{1,2,\ldots,N\}} \left|\langle \mathbf{r}^{k-1}, \varphi_j \rangle\right|$ and complementary candidate set C^k_a by using $\mathbf{r}^{k-1}_a$ (see (3)).

2 Atoms' addition and detection: let $S^k = T^{k-1} \cup C^k \cup C^k_a$, compute $\mathbf{x}^k_{S^k} = \mathbf{\Phi}^\dagger_{S^k}\mathbf{y}$.

3 Update: obtain the estimated support set T^k, whose element j satisfies: $\left|x^k_j\right| \ge \mu_2 \cdot \max\left|\mathbf{x}^k_{C^k}\right|$, and update estimated signal $\mathbf{x}^k_{T^k} = \mathbf{\Phi}^\dagger_{T^k}\mathbf{y}$ and residue $\mathbf{r}^k = \mathbf{y} - \mathbf{\Phi}_{T^k}\mathbf{x}^k_{T^k}$.

4 if $\left\|\mathbf{r}^k\right\|_2 < \varepsilon$ or $k = k_{\max}$, quite the iteration, otherwise compute auxiliary estimated support set Γ^k (see (2)) and corresponding residual $\mathbf{r}^k_a = \mathbf{y} - \mathbf{\Phi}_{\Gamma^k}\mathbf{\Phi}^\dagger_{\Gamma^k}\mathbf{y}$, then let $k = k+1$ and go to step 1.

Output: estimated support set T^k and estimated signal

$$\hat{\mathbf{x}} = \begin{cases} \mathbf{\Phi}^\dagger_{T^k}\mathbf{y} & \text{on the support set } T^k \\ \mathbf{0} & \text{elsewhere} \end{cases}$$

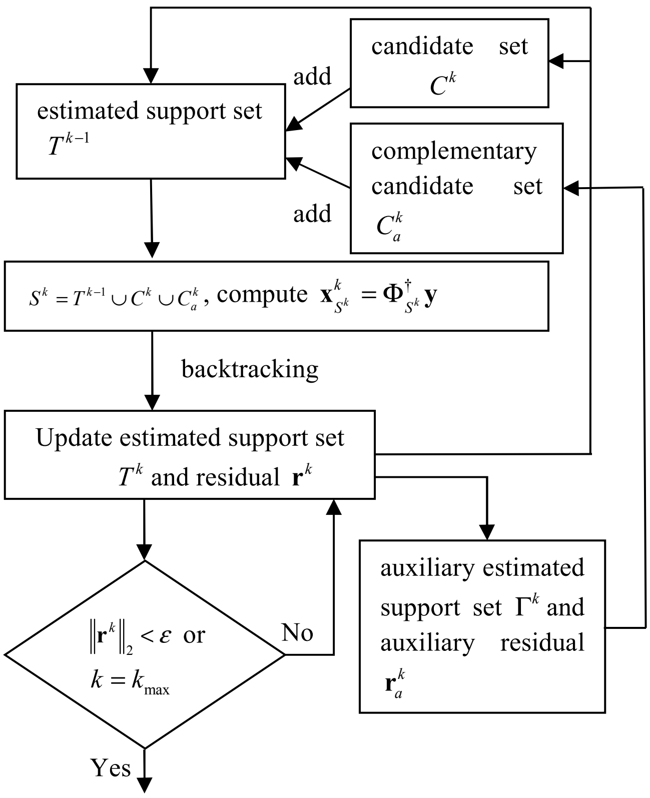


Fig. 1 Flow chart of AR-AMP algorithm.

In the proposed AR-AMP algorithm, we still use the backtracking strategy of the BAOMP to discard the atoms whose coefficients are smaller than that of the currently chosen atoms, which has the largest correlation with current residual. This proposed algorithm can be regarded as an improved BAOMP algorithm, which utilizes auxiliary residual to choose penitential correct atoms.

Consider the construction of auxiliary estimated support set Γ^k and complementary candidate set C_a^k, here their sizes are set by the fix scale technique of SP. Since the sparsity is unknown, we use the OMP's strategy and let the sizes of the two sets be equal to iteration number, which implies that it will grow with the increase of iteration. With grow of iteration, the sizes of Γ^k and C_a^k will become large and more atoms will be added. Consider this point, to maintain the advantage of the BAOMP, if the size of Γ^k is bigger than that of the estimated support set T^k, the former is required to be equal to the latter. That is to say, the auxiliary estimated support set Γ^k is

$$\Gamma^k = \begin{cases} \{i_1,\dots,i_k\} = \arg\max\limits_{card(I)=k} \left\| \left(\mathbf{\Phi}^{\dagger}_{T^k}\mathbf{y}\right)_I \right\|_2^2 & \text{if } k<card\left(T^k\right) \\ T^k & \text{elsewhere} \end{cases} \quad (2)$$

At the same time, the complementary candidate set C_a^k is

$$C_a^k = \begin{cases} C^k & \text{if } \mathbf{r}_a^{k-1} = \mathbf{r}^{k-1} \\ \{j_1,\dots,j_k\} = \arg\max\limits_{card(J)=k} \left\| \left(\mathbf{\Phi}^{*}\mathbf{r}_a^{k-1}\right)_J \right\|_2^2 & \text{elsewhere} \end{cases} \quad (3)$$

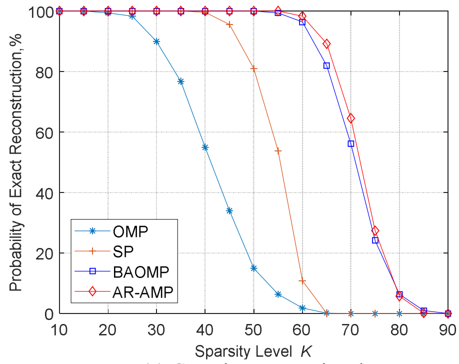
It can be seen that the AR-AMP will reduce to the BAOMP when the number of iteration is large enough.

III. EXPERIMENTAL RESULTS

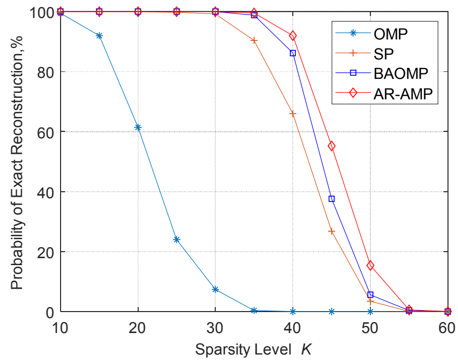
A. Simulation for Synthetic Data

In this subsection some experiments are made to demonstrate the performance of the AR-AMP and comparison with the OMP, SP, BAOMP is also given. The OMP uses K iterations. In the SP, $k_{\max} = 100$. The parameters of the BAOMP and AR-AMP are the same, i.e., $\mu_1 = 0.4, \mu_2 = 0.6$, $\varepsilon = 10^{-6}$, $k_{\max} = 100$. In the experiment, the signals of interests are Gaussian or binary sparse signals. The support set is selected uniformly at random and each nonzero value is generated from a standard Gaussian distribution or one for the binary signal. The measurement matrix $\mathbf{\Phi}$ is generated randomly from the standard i.i.d. Gaussian distribution in each trial, and all of its columns are scaled to unit norm.

To evaluate the reconstruction performances of different algorithms, the probabilities of exact recovery are calculated. The reconstruction is considered to be exact when the magnitude difference between the original signal $\mathbf{x}$ and the reconstructed one $\hat{\mathbf{x}}$ is smaller than 10^{-3}, that is, $\max\limits_{i=1,2,\cdots,N}\left(\left|x_i - \hat{x}_i\right|\right) < 10^{-3}$.



(a) Gaussian sparse signal



(b) Binary sparse signal

Fig. 2 Probability of exact reconstruction vs. signal sparsity.

The length of $\mathbf{x}$ is $N = 256$ and the size of $\mathbf{\Phi}$ is 128×256. The probabilities of exact recovery at different sparsity are calculated and 500 independent simulations are performed for each sparsity. The experiment result is given in Fig. 2 and it shows that AR-AMP is better than other greedy algorithms, especially for the binary sparse signals.

To demonstrate computation complexity of the AR-AMP, for binary sparse signals, Fig.3 compares the average running time of these greedy algorithms versus the signal sparsity, where all algorithms are tested by Matlab R2018b with Intel Core i7 processor, 8 GB memory and Windows 7 environment. It can be seen that the proposed AR-AMP is a litter more time-consuming than the BAOMP when the sparsity is small, this is due to the additional computation of auxiliary residual. When the sparsity is from 35 to 50, the time-consuming of AR-AMP is a little less than that of the BAOMP while having better success reconstruction rate.

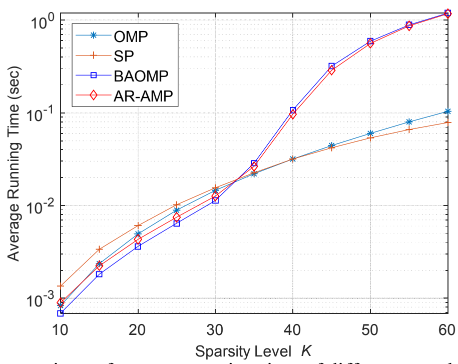
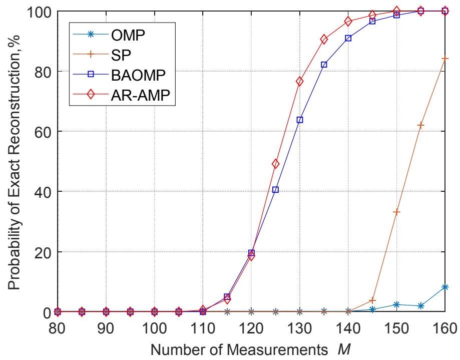
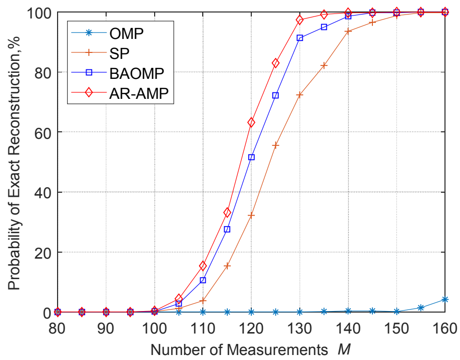


Fig. 3 Comparison of average running time of different greedy algorithms.



(a) Gaussian sparse signal



(b) Binary sparse signal

Fig. 4 Probability of exact reconstruction vs. the number of measurements.

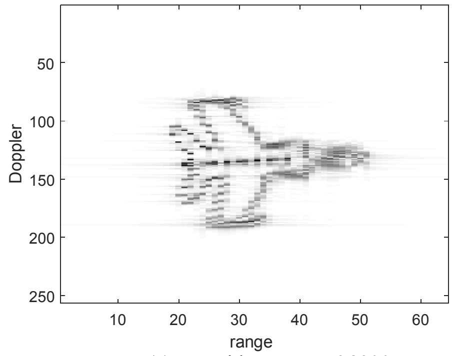
Then given the sparsity, the relationship between the number of measurements and the probability of exact reconstruction is observed. The number of measurements M is chosen from 80 to 160 and 500 trial are proceeded for each M . Fig. 4 demonstrates the simulation results, where the signal sparsity $K=70$ for Gaussian signal and $K=40$ for binary sparse signal. It is shown that if the reconstruction rate is the same, AR-AMP needs minimum number of measurements compared with other greedy algorithms when the original signal is not sparse enough.

From simulations it can be seen that, no matter Gaussian signal or binary signal, the performance of the AR-AMP is better than that of other greedy algorithms.

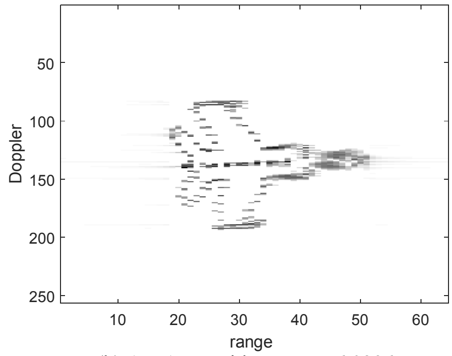
B. Simulation for CS-based ISAR Imaging

Recently CS-based radar imaging techniques which utilize the sparsity of targets have been demonstrated to be promising methods for high-resolution radar imaging. Some sparse transforms have been used, such as the Fourier transform [16-18] and fractional Fourier transform (FrFT) [19]. Here, to further demonstrate the performance of the AR-AMP algorithm in practical application, inverse synthetic aperture radar (ISAR) imaging is considered within the sparse signal recovery framework.

In this experiment we use the simulated MIG-25 data. The stepped frequency radar operates at 9GHz and has a bandwidth of 512MHz. For each pulse, 64 complex range samples were saved. The pulse repetition frequency is 15KHz. Consider the characteristic of the echo signal which can be modeled as the linear frequency modulated (LFM) signal [20,21], the FrFT matrix is used as the sparsifying matrix. The parameters of the AR-AMP are $\mu_1=0.5, \mu_2=1$, $\varepsilon=10^{-6}$, $k_{\max}=16$. Image entropy is utilized to measure the quality of ISAR images.

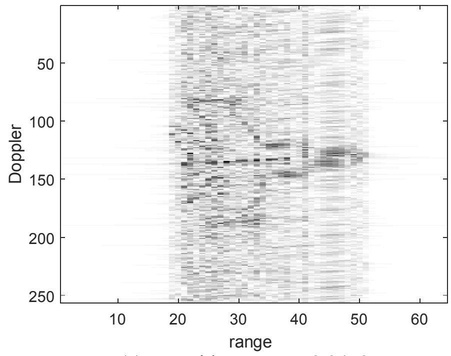


(a) RD with entropy=6.8900

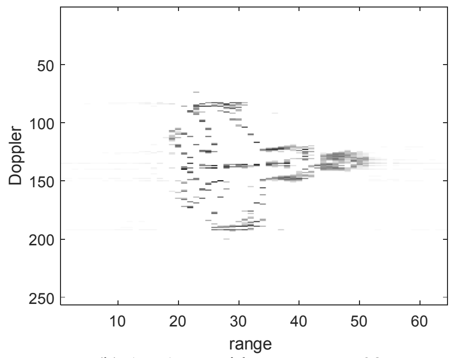


(b) AR-AMP with entropy=6.0296

Fig. 5 ISAR images by different algorithms with full pulses.



(a) RD with entropy= 8.3478



(b) AR-AMP with entropy=5.7987

Fig. 6 ISAR images by different algorithms under sub-sampling.

Fig. 5 and Fig. 6 give the images obtained by the RD method and AR-AMP algorithm with full 256 pulses and randomly sampled 76 pulses, respectively. From these imaging results, it can be seen that the image produced by traditional RD technique will deteriorate under sub-sampling, while the AR-AMP algorithm can still generate good images.

IV. Conclusions

In this paper, an auxiliary residual based adaptive matching pursuit (AR-AMP) algorithm is proposed, which is based on BAOMP and can choose possible correct atoms in each iteration by using auxiliary residual to make correlation test. Simulations have showed that the AR-AMP has improved recovery performance. In AR-AMP, design of auxiliary estimated support set and complementary candidate set can use other techniques, future work will focus on finding better design methods to further improve the performance of such an auxiliary residual based greedy method.

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