Improvement of Estimation Accuracy of Wideband DOA Estimation "TOPS"

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Abstract—**In this paper, a new direction-of-arrival (DOA) estimation method for wideband signals is introduced. This new method is based on test of orthogonality of projected subspaces (TOPS) and estimated DOAs by measuring the orthogonal relation between the signal and noise subspaces of multiple frequency components of the signals. Unlike other coherent wideband methods, such as the coherent signal subspace method (CSSM), the new method does not require any preprocessing for initial values of DOA, like TOPS. The resolution and root mean square error (RMSE) of the proposed method is compared with those of CSSM, TOPS and test of orthogonality of frequency subspaces (TOFS) through computer simulations. The simulation results show that the resolution of the proposed method is better than that of TOPS in low and mid signal-to-noise ratio (SNR) ranges. They also show that RMSE of the proposed method is better than that of TOPS in mid SNR ranges.**

I. INTRODUCTION

Several techniques have been proposed to solve the problem of estimating direction-of-arrivals (DOAs) of multiple wideband signals [1]. Many narrowband methods are not applicable to wideband signals directly. The reason is that the energies of narrowband signals are concentrated in a frequency band that is relatively small compared with the center frequency. Thus, the sensor output can be easily vectorized by using one frequency component for narrowband signal. For wideband signals, the phase can be easily vectorized by using one frequency component. For wideband signals, the phase difference between sensor outputs depends on not only DOA but also the temporal frequency [2].

The common method for processing wideband signal sources is decomposing the wideband signals into a set of narrowband signals with different frequencies by discrete Fourier transform (DFT). Based on this method, many algorithms were introduced. The incoherent signal subspace method (ISSM) is one of the simplest wideband DOA estimation methods [3][4]. ISSM uses some frequency components incoherently. It simply averages the results of some frequency components, which are processed by narrowband techniques, such as MUSIC (Multiple Signal Classification). IMUSIC is ISSM that uses MUSIC [5] in narrowband methods. Although ISSM is simple and effective in high SNR, it suffers when the SNR at each frequency varies, because the DOA estimates at some frequencies may be very bad. A single outlier could spoil the final estimates in the averaging process.

To overcome such disadvantages, a number of improved methods have been proposed. Those are coherent signal subspace method (CSSM) [6] and weighted average of signal subspaces (WAVES) [7]. The CSSM method requires initial DOAs, and its performance is sensitive to these initial values. WAVES also requires initial values.

The test of orthogonality of projected subspace (TOPS) [8] is a newly introduced method, and is referred to as a noncoherent method. This method first uses singular value decomposition (SVD) to get the signal subspace of one frequency point, which belongs to the bandwidth of every signal. This signal subspace of one frequency and one DOA can be transformed to another frequency and another DOA, respectively. Then, the orthogonality of transformed signal subspaces and noise subspaces of every DOA and every frequency is tested.

The test of orthogonality of frequency subspaces (TOFS) [9] is also a new incoherent method, like ISSM. TOFS constructs the searching steering vectors of every possible DOA and every frequency. Since TOFS is an incoherent method, estimation accuracy is very high when SNR is high. However, TOFS cannot resolve the desired DOAs when SNR is low.

In this paper, we propose a new wideband DOA estimation based on TOPS. The proposed method uses signal subspace, like TOPS. However, the proposed method uses signal subspace twice and tests squared the orthogonality of signal subspaces and noise subspaces. As a result, the orthogonality of projected signal subspaces and noise subspaces of every DOA and every frequency is tested on both rows and columns of submatrices of the matrix composed for test of orthogonality. For example, when the lth wave arrives, the lth row and the

lth column elements of submatrices of the matrix composed for test of orthogonality approach 0. That is, the lth wave is orthogonal to the noise subspaces. We show that the resolution of the proposed method performs better than that of the others except CSSM in low SNR ranges and better than that of TOPS in mid SNR ranges. The simulation results also show that RMSE of the proposed method is better than that of TOPS in high SNR ranges.

The advantage of the proposed method is that the resolution has been improved from that of TOFS and TOPS in low SNR, and RMSE has been improved from that of TOPS in high SNR.

II. Narrowband Subspace Method

The subspace method is an electric wave propagation analysis technique using the characteristics of the eigenvalue and the eigenvector obtained from the eigenvalue decomposition (EVD) to data correlation matrix. The feature of the subspace method is to be able to separate a signal subspace from a noise subspace. Thus, the subspace method is applied to wideband DOA estimation.

Correlation matrix $\mathbf{R}_{xx}$ is as follows.

$$\mathbf{R}_{xx}\mathbf{v}_i = (\mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma^2\mathbf{I})\mathbf{v}_i = \lambda_i\mathbf{v}_i, \qquad i = 1, 2, \cdots, M \tag{1}$$

where M is the number of elements, $\mathbf{A}$ is the $M \times L$ steering matrix

$$\mathbf{A}(\theta) = [a(\theta_i), a(\theta_2), \cdots, a(\theta_L)] \tag{2}$$

whose columns are the $M \times 1$ array manifolds, and L is the number of incoming waves. Eigenvalue λ_i and its corresponding eigenvector $\mathbf{v}_i$ can be formed from the EVD

$$\begin{aligned} \mathbf{R}_{xx} &= \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma^2\mathbf{I} \\ &= \sum_{i=1}^{M} \lambda_i \mathbf{v}_i \mathbf{v}_i^H && (3) \\ &= \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H && (4) \end{aligned}$$

where

$$\begin{aligned} \mathbf{V} &= [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_M] && (5) \\ [\mathbf{\Lambda}]_{(m,m)} &= \lambda_m && (6) \end{aligned}$$

$[\mathbf{\Lambda}]_{(m,m)}$ is the mth element of the diagonal matrix. $\mathbf{R_{xx}}$ is a positive definite Hermitian matrix. Thus, the eigenvalue is a positive real number

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M (> 0) \tag{7}$$

The next expression holds from eq. (1).

$$\begin{aligned} \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H\mathbf{v}_i = (\lambda_i - \sigma^2)\mathbf{v}_i &= \lambda_i'\mathbf{v}_i, && (8) \\ i &= 1, 2, \cdots, M && (9) \\ \lambda_i' &= \lambda_i - \sigma^2 && (10) \end{aligned}$$

The signal of a coherent wave group is considered to be one wave when thinking about the rank of the matrix $\mathbf{ASA}^H$, and the rank is deficient. The sum of the number of coherent wave groups and incoherent waves becomes the rank of $\mathbf{ASA}^H$.

Since the K+1 th and above eigenvalues are 0, the distribution of the eigenvalues are as follows.

$$\lambda_1' \geq \lambda_2' \geq \cdots \geq \lambda_L' > \lambda_{L+1}' = \cdots = \lambda_M' = 0 \tag{11}$$

The eigenvalues of the receive data correlation matrix $\mathbf{R}_{xx}$ are distributed as follows.

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L > \lambda_{L+1} = \cdots = \lambda_M = \sigma^2 \tag{12}$$

The distribution of the eigenvalues of the receive data correlation matrix $\mathbf{R}_{xx}$ can be divided into L signal eigenvalues corresponding to the sum of the number of coherent wave groups and incoherent waves, and be divided into the $M - L$ noise eigenvalues equal to the noise power.

Next, the characteristics of the eigenvector are shown. From eq. (1),

$$\mathbf{R}_{xx}\mathbf{v}_i = (\mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma^2\mathbf{I})\mathbf{v}_i = \lambda_i\mathbf{v}_i = \sigma^2\mathbf{v}_i, \quad i = L+1, L+2, \cdots, M \tag{13}$$

Thus,

$$\mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H\mathbf{v}_i = \mathbf{0}, \; i = L+1, L+2, \cdots, M \tag{14}$$

That is,

$$\mathbf{a}(\theta_l)^H\mathbf{v}_i = 0, \; i = L+1, L+2, \cdots, M, \; l = 1, 2, \cdots, L \tag{15}$$

Therefore, the eigenvector corresponding to the noise eigenvalue is orthogonal to a new steering vector with which the steering vector of a coherent wave group and an incoherent wave is a linear combination.

The receive data correlation matrix $\mathbf{R}_{xx}$ uses the above-mentioned eigenvalues and the eigenvectors, and can be represented as follows.

$$\begin{aligned} \mathbf{R}_{xx} &= \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma^2\mathbf{I} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H \\ &= \mathbf{F}\mathbf{\Lambda}_S\mathbf{F}^H + \mathbf{W}\mathbf{\Lambda}_N\mathbf{W}^H && (16) \\ \mathbf{F} &= [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_L] && (17) \\ \mathbf{W} &= [\mathbf{v}_{L+1}, \mathbf{v}_{L+2}, \cdots, \mathbf{v}_M] && (18) \\ \mathbf{\Lambda}_S &= \mathrm{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_L\} && (19) \\ \mathbf{\Lambda}_N &= \mathrm{diag}\{\lambda_{L+1}, \lambda_{L+2}, \cdots, \lambda_M\} && (20) \end{aligned}$$

The linear space that the M eigenvectors of receive data correlation matrix $\mathbf{R}_{xx}$, put is divided into two subspaces such as a signal subspace by the signal eigenvector $\mathcal{S}$ and a noise subspace $\mathcal{N}$ by the noise eigenvectors, respectively as follows.

$$\begin{aligned} \mathcal{S} &= \mathrm{span}\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_L\} && (21) \\ \mathcal{N} &= \mathrm{span}\{\mathbf{v}_{L+1}, \mathbf{v}_{L+2}, \cdots, \mathbf{v}_M\} && (22) \end{aligned}$$

where span is a set of vectors that consist of a linear coupling vectors.

The matrices projected to a signal subspace and a noise subspace are as follows.

$$\begin{aligned} \mathbf{P}_S &= \mathbf{F}\mathbf{F}^H = \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H && (23) \\ \mathbf{P}_N &= \mathbf{W}\mathbf{W}^H = \mathbf{I} - \mathbf{P}_S = \mathbf{I} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H && (24) \end{aligned}$$

III. MUSIC METHOD

In the MUSIC method [5], the orthogonalization of a signal subspace and the noise subspace obtained from EVD of the correlation matrix is used. From correlation matrix, $M-L$ space spectrum can be defined as follows by the use of the noise eigenvectors obtained from EVD of the correlation matrix.

$$P_{MUSIC}^{(i)}(\theta)=\frac{\mathbf{a}(\theta)^H\mathbf{a}(\theta)}{|\mathbf{a}(\theta)^H\mathbf{v}_i|^2},\quad i=L+1,L+2,\cdots,M \tag{25}$$

There is a possibility that a spurious peak is caused, though all these spatial spectra are diverged with $\theta=\theta_l,\ l=1,2,\cdots,$ and L.

If the average processing is done, a spurious peak can be suppressed, because the angle where a spurious peak is caused is different in general. The average processing of spectrum based on the following harmonic averages is introduced in the MUSIC method, and it is defined as follows.

$$\begin{aligned}&P_{MUSIC}(\theta)\\ &=\frac{1}{\frac{1}{P_{MUSIC}^{(K+1)}(\theta)}+\frac{1}{P_{MUSIC}^{(K+2)}(\theta)}+\cdots+\frac{1}{P_{MUSIC}^{(M)}(\theta)}} &(26)\\ &=\frac{\mathbf{a}(\theta)^H\mathbf{a}(\theta)}{\sum_{i=L+1}^{M}|\mathbf{a}(\theta)^H\mathbf{v}_i|^2} &(27)\\ &=\frac{\mathbf{a}(\theta)^H\mathbf{a}(\theta)}{\mathbf{a}(\theta)^H\mathbf{W}\mathbf{W}^H\mathbf{a}(\theta)} &(28)\end{aligned}$$

In the MUSIC method, the DOA estimation is done using this spectrum.

IV. CONVENTIONAL WIDEBAND SUBSPACE METHODS

The wideband signal is decomposed into two or more narrowband signals by DFT in many wideband DOA estimation [2]. Wideband DOA estimation is done by applying the narrowband DOA estimation to the decomposed narrowband signals.

Section IV-A, IV-B, V, and VI explain the conventional wideband DOA estimation.

A. From Wideband Signals to Narrowband Signals

It is assumed that all signal sources exist in bandwidth $[\omega_L,\ \omega_H]$. At that time, the wideband signal cannot be approximated to the narrowband signal as described in Section II. Then, the wideband signal is decomposed into two or more narrowband signals by DFT.

The DFT is

$$X_m(\omega)=\sum_{l=1}^{L}S_l(\omega)\exp\left(-j\omega\frac{md}{c}\sin\theta_l\right)+N_m(\omega) \tag{29}$$

where $\omega=2\pi f$. Therefore, the received signal of all elements, eq. (29), is divided into K frequencies by DFT as follows.

$$\mathbf{X}(\omega_i)=\mathbf{A}(\omega_i,\theta)\mathbf{S}(\omega_i)+\mathbf{N}(\omega_i),\quad i=1,2,\cdots,K \tag{30}$$

where $\omega_L<\omega_i<\omega_H$. Thus, the signal can be approximated to the narrowband signals described in Section II.

As a result, the correlation matrix can be described as follows.

$$\begin{aligned}\mathbf{R}_i &= \mathbf{E}[\mathbf{x}_i(t)\mathbf{x}_i(t)^H] &(31)\\ &= \mathbf{A}_i\mathbf{R}_{ss}(\omega_i)\mathbf{A}_i^H+\sigma^2\mathbf{I} &(32)\\ &= \mathbf{F}_i\mathbf{\Lambda}_S(\omega_i)\mathbf{F}_i^H+\mathbf{W}_i\mathbf{\Lambda}_N(\omega_i)\mathbf{W}_i^H &(33)\\ \mathbf{F}_i &= [\mathbf{v}_{i,1},\mathbf{v}_{i,2},\cdots,\mathbf{v}_{i,L}] &(34)\\ \mathbf{W}_i &= [\mathbf{v}_{i,L+1},\mathbf{v}_{i,L+2},\cdots,\mathbf{v}_{i,M}] &(35)\end{aligned}$$

where $\mathbf{F}_i$ is a signal subspace of frequency ω_i and $\mathbf{W}_i$ is noise subspace of frequency ω_i.

The following conventional methods decompose the wideband signal into two or more narrowband signals.

B. Incoherent Wideband DOA Estimation

An incoherent method independently applies the narrowband subspace method to the received signal corresponding to each frequency.

C. IMUSIC (Incoherent MUSIC)

IMUSIC applies the narrowband signal subspace method, MUSIC, to the correlation matrix of each frequency independently [3][4][5].

$$\hat{\theta}=\arg\min_{\theta}\sum_{i=1}^{K}\mathbf{a}^H(\omega,\theta)\mathbf{W}\mathbf{W}^H\mathbf{a}(\omega,\theta) \tag{36}$$

Features of IMUSIC

- Advantage: Estimation accuracy in high SNR is high.
- Disadvantage: If the estimation result of a frequency bin is bad, the bad result influences final estimation, because it averages the result of each frequency to set the final result, even in high SNR.

D. TOFS (Test of Orthogonality of Frequency Subspace)

TOFS uses the noise subspace obtained from EVD of the correlation matrix of each frequency [9]. The DOA is estimated by using the evaluation function of MUSIC [5], $\mathbf{a}^H(\omega,\theta)\mathbf{W}\mathbf{W}^H\mathbf{a}(\omega,\theta)$. When θ is the DOA, that is, $\hat{\theta}=\theta$, this evaluation function fulfills the following equation.

$$\mathbf{a}^H(\omega,\hat{\theta})\mathbf{W}\mathbf{W}^H\mathbf{a}(\omega,\hat{\theta})=0 \tag{37}$$

The following matrix, $\mathbf{D}$ is defined by using this evaluation function.

$$\begin{aligned}\mathbf{D}(\theta) = &[\mathbf{a}^H(\omega_1,\theta)\mathbf{W}_1\mathbf{W}_1^H\mathbf{a}(\omega_1,\theta)\mid\mathbf{a}^H(\omega_2,\theta)\mathbf{W}_2\mathbf{W}_2^H\mathbf{a}(\omega_2,\theta)\mid\\ &\cdots\mid\mathbf{a}^H(\omega_K,\theta)\mathbf{W}_K\mathbf{W}_K^H\mathbf{a}(\omega_K,\theta)] \end{aligned}\tag{38}$$

As for this matrix $\mathbf{D}$, the rank decreases when $\hat{\theta}$ is a DOA, that is, $\hat{\theta}=\theta$. The DOA is estimated from the following equation by using a decrease in the rank of this matrix $\mathbf{D}$.

$$\hat{\theta}=\arg\max_{\theta}\frac{1}{\sigma_{\min}(\theta)} \tag{39}$$

where $\sigma_{min}(\theta)$ is a minimum singular value of the matrix, $\mathbf{D}$.

Features of TOFS

- Advantage: Resolution in low SNR is improved from IMUSIC by using a decrease in the rank of the matrix.
- Advantage: Estimation accuracy in high SNR is high.
- Disadvantage: Resolution in low SNR is low.

V. Coherent Wideband DOA Estimation

A. CSSM (Coherent Signal Subspace Method)

CSSM transforms the correlation matrices at many frequencies into one general correlation matrix at one frequency (reference frequency or focusing frequency) by using a transformation matrix (focusing matrix) that depends on a frequency bin [6]. This procedure is referred to as focusing. The MUSIC method is applied to the average correlation matrix for the average of the transformed correlation matrices.

The DOA estimation procedure of CSSM is as follows.

First, the correlation matrices at many frequencies are transformed into one general correlation matrix at the focusing frequency, (focusing). Initial estimation in the DOA is necessary to make the focusing matrix. This initial estimation is referred to as a focusing angle. The focusing matrix can be obtained from the following equation [10].

$$\min_{\mathbf{T}_i} ||\mathbf{A}(\omega_1, \boldsymbol{\theta}_f) - \mathbf{T}_i\mathbf{A}(\omega_i, \boldsymbol{\theta}_f)||_F, \ i = 2, \cdots K \tag{40}$$

where $\mathbf{T}_i$ is the focusing matrix at frequency ω_i, ω_1 is the focusing frequency, and $\boldsymbol{\theta}_f$ is the focusing angle. The averaging procedure of the correlation matrices is done as follows.

$$\mathbf{R}_{gen} = \frac{1}{K}\sum_{i=1}^{K} \mathbf{T}_i\mathbf{R}_i\mathbf{T}_i^H, \ i = 1, \cdots K \tag{41}$$

where the focusing matrix, $\mathbf{T}_1$, is a unit matrix, if focusing frequency is ω_1.

As shown above, the MUSIC method is applied to generate the average correlation matrix $\mathbf{R}_{gen}$, and the DOA is estimated. The estimation accuracy of CSSM depends on the focusing angle greatly. As a result, an inaccurate focusing angle cannot transform frequencies accurately because of the error in the focusing matrix. Therefore, we have to be able to do focusing accurately.

Features of CSSM

- Advantage: DOA estimation can be performed by focusing even in low SNR.
- Disadvantage: Estimation accuracy greatly depends on focusing angle regardless of SNR.
- Disadvantage: Initial estimation is needed for focusing, and the computational complexity for that increases.

VI. Noncoherent Wideband DOA Estimation

The noncoherent method exploits the signal and noise subspaces of multiple frequencies. Thus, the noncoherent method is different from the incoherent method and the coherent method. The noncoherent method is different from the usual incoherent methods, since it takes advantage of subspaces from multiple frequencies simultaneously. It is also different from the coherent methods that form a general coherent correlation matrix using focusing angles. Thus, the noncoherent method sits between incoherent and coherent methods. Also, the performance sits between incoherent and coherent methods.

A. TOPS (Test of Orthogonality of Projected Subspace)

TOPS [8] exploits the signal and noise subspaces of multiple frequencies. The DOA estimation procedure of TOPS is as follows.

First of all, the signal subspace $\mathbf{F}_i$ and the noise subspace $\mathbf{W}_i$ are obtained from EVD of the correlation matrix of each frequency. Next, the frequency of one signal subspace is transformed into the other frequencies. This frequency transform is different from focusing in CSSM, and the frequency transform of a signal subspace in TOPS does not need initial estimation of DOA. The matrix $\Phi(\omega_i, \theta)$, used for the frequency transform of a signal subspace in TOPS, is a diagonal matrix, and the element of the mth row and the mth column of the diagonal matrix is

$$[\mathbf{\Phi}(\omega_i, \theta)]_{(m,m)} = \exp\left(-j\omega_i \frac{md}{c}\sin\theta_i\right) \tag{42}$$

This matrix $\Phi(\omega_i, \theta)$ is used for the frequency transform of a signal subspace.

$$\begin{aligned}\mathbf{U}_j(\hat{\phi}) &= \mathbf{\Phi}(\Delta\omega, \phi)\mathbf{F}_i && (43)\\ &= \mathbf{\Phi}(\Delta\omega, \phi)\mathbf{A}_i(\hat{\boldsymbol{\theta}})\mathbf{G}_i && (44)\\ &= \mathbf{A}_j(\hat{\boldsymbol{\theta}})\mathbf{G}_i && (45)\end{aligned}$$

where $\mathbf{U}_j(\theta)$ is a signal subspace at frequency ω_j, $\Delta\omega = \omega_j - \omega_i$, $\mathbf{F}_i$ is a signal subspace at the frequency ω_i, $\mathbf{A}_i(\hat{\theta})$ is a steering matrix, and $\hat{\theta}$ is a DOA. $\mathbf{G}_i$ is the $L \times L$ square matrix. It is noticed that only the steering matrix at the frequency ω_i is transformed from the frequency ω_i to the frequency ω_j.

Assume that the signal subspace $\mathbf{F}_1$ at the frequency ω_1 is transformed from the frequency ω_1 to the frequencies $\omega_2 \sim \omega_K$. The transformed signal subspaces and the noise subspaces obtained previously compose the following matrix, $\mathbf{D}$.

$$\mathbf{D}(\theta) = [\mathbf{U}_2^H\mathbf{W}_2 \mid \mathbf{U}_3^H\mathbf{W}_3 \mid \cdots \mid \mathbf{U}_K^H\mathbf{W}_K] \tag{46}$$

As for this matrix, $\mathbf{D}$, the rank decreases when θ is a DOA, that is, $\hat{\theta} = \theta$. The DOA is estimated from the following equation by using a decrease in the rank of this matrix, $\mathbf{D}$.

$$\hat{\theta} = \arg\max_{\theta} \frac{1}{\sigma_{\min}(\theta)} \tag{47}$$

where $\sigma_{min}(\theta)$ is a minimum singular value of the matrix $\mathbf{D}$.

B. Features of TOPS

- Advantage: DOA estimation is almost possible in low SNR.
- Disadvantage: Even in high SNR a few error occurs in estimation.

VII. Proposed Method

A. Problem of Conventional Wideband DOA Estimation

Conventional wideband DOA estimations, IMUSIC, TOFS, and TOPS are low resolution in low SNR. Although CSSM is high resolution in low SNR, CSSM needs a lot of computational complexities. As for RMSE of TOPS, a few estimation error occurs in mid and high SNR.

B. Proposed Method

We propose a method that improves TOPS. The reasons why the proposed method is based on TOPS are as follows.

- DOA estimation can be resolved in low SNR, because TOPS uses a signal subspace directly.
- Initial estimation of DOA necessary for focusing of CSSM is unnecessary.

Then, we propose the DOA estimation method to aim at the improvement of the estimation accuracy in mid and high SNR and at the improvement of resolution in low SNR without initial estimation of DOA. The process of the proposed method is as follows.

C. Process of Proposed Method

A signal subspace of one frequency is transformed into the other frequencies as well as TOPS. It is assumed that the signal subspace $\mathbf{F}_1$ of the frequency ω_1 is transformed to the frequencies $\omega_2 \sim \omega_K$.

After the frequency transforming, we compose the matrix $\mathbf{Z}(\phi)$ to perform the squared test of orthogonality of signal subspaces and noise subspaces. The matrix $\mathbf{Z}(\phi)$ is

$$\begin{aligned}\mathbf{Z}(\phi) \quad = \quad & [\mathbf{U}_2^H \mathbf{W}_2 \mathbf{W}_2^H \mathbf{U}_2 \mid \mathbf{U}_3^H \mathbf{W}_3 \mathbf{W}_3^H \mathbf{U}_3 \mid \\ & \cdots \mid \mathbf{U}_K^H \mathbf{W}_K \mathbf{W}_K^H \mathbf{U}_K] \end{aligned} \tag{48}$$

Signal subspaces transformed from the frequency ω_1, $\mathbf{U}_2 \sim \mathbf{U}_K$, satisfy the following equation.

$$\mathbf{U}_j = \mathbf{A}_j \mathbf{G}_1, \quad j = 2, 3, \ldots, K \tag{49}$$

From this equation,

$$\mathbf{U}_j^H \mathbf{W}_j \mathbf{W}_j^H \mathbf{U}_j = \mathbf{G}_1^H \mathbf{A}_j^H(\hat{\boldsymbol{\theta}}) \mathbf{W}_j \mathbf{W}_j^H \mathbf{A}_j(\hat{\boldsymbol{\theta}}) \mathbf{G}_1 \tag{50}$$

As well as TOPS, the DOA is estimated by using a decrease in the rank of this matrix, $\mathbf{Z}(\phi)$.

VIII. Simulation Results

A. Simulation Model

The proposed method has been tested through computer simulations by considering a ten-sensor uniform linear array (ULA) with inter element distance d equal to $\lambda/2$, where λ is the wavelength corresponding to the maximum frequency component of the received signals. The statistical performance was evaluated by performing 1000 Monte Carlo runs for each scenario. Three far-field uncorrelated wideband signals are placed at $8°$, $33°$ and $37°$, respectively. The sampling frequency is three times the highest frequency. The wideband signal sits between $\pi/3$ and $2\pi/3$ in the ω domain.

The proposed method was compared to the other methods: CSSM in coherent wideband DOA estimation, TOFS in incoherent wideband DOA estimation, and TOPS in noncoherent wideband DOA estimation. For example, seven frequency bins are used in the proposed method and all the conventional methods. RSS focusing matrices are used to correlate the sample covariance matrices in CSSM. Linear Prediction (LP) method is employed as the low-resolution algorithm. DOA estimates of multiple frequency bins by the LP method are averaged and used as a focusing angle in the CSSM. Results are shown in Figs. 1, 2, 3, and 4.

B. Results

Fig. 1 shows the probability of resolution in each SNR, where the probability of resolution is defined as the probability that was able to separate the desired incoming waves. We can see that the proposed method improves the resolution than TOPS in low and medium SNR. There are two reasons why the resolution of the proposed method is improved in low and medium SNR. First, the proposed method correlates the signal subspace of each frequency by the frequency transformation. Second, the proposed method uses the following matrix as a submatrix to perform the squared test of orthogonality of signal subspaces and noise subspaces. The submatrix of the proposed method is

$$\mathbf{U}_j^H \mathbf{W}_j \mathbf{W}_j^H \mathbf{U}_j = \mathbf{G}_1^H \mathbf{A}_j^H(\hat{\theta}) \mathbf{W}_j \mathbf{W}_j^H \mathbf{A}_j(\hat{\theta}) \mathbf{G}_1 \tag{51}$$

When θ is a DOA, that is, $\hat{\theta} = \theta$, the row and column elements of the submatrix approach 0 in the proposed method. In TOPS, only the row elements of the submatrix approach 0. As a result, the resolution of the proposed method improves in low and medium SNR.

Figs. 2, 3, and 4 show RMSE at $8°$, $33°$ and $37°$, respectively. The proposed method decreases the DOA estimation error more than TOPS in mid SNR. In addition, the DOA estimation error of the proposed method is the same as that of TOPS in high SNR. The proposed method tests the orthogonality of the signal subspace and the noise subspace by both the row and column elements of the submatrix as described before. That is, both the row and the column elements approach 0. Furthermore the diagonal element of the submatrix that is the product of the row and the column approaches 0 more than that of TOPS.

IX. Conclusion

We propose the DOA estimation for wideband signals aiming at the improvement of the estimation accuracy in mid and high SNR based on the DOA estimation for wideband signals, "TOPS". The proposed method estimates DOAs by measuring the orthogonal relation between the signal and noise subspaces of multiple frequency components of the signals. Unlike CSSM, the proposed method does not require any preprocessing for initial values of DOA. The proposed method uses the signal subspaces and noise subspaces twice and performs the squared test of orthogonality of signal subspaces and

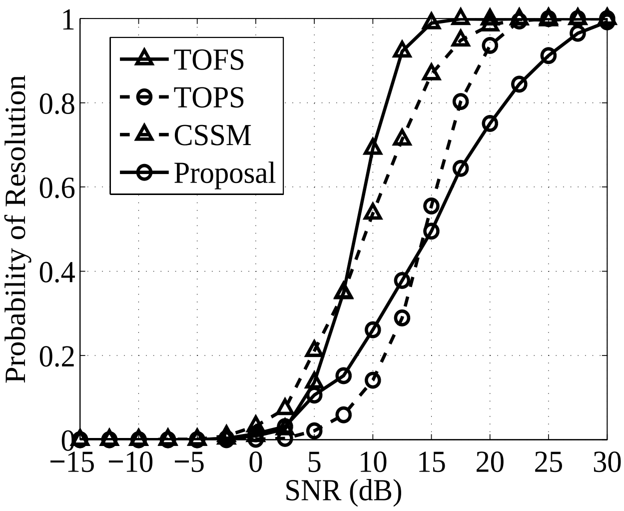

Fig. 1. SNR vs probability of resolution

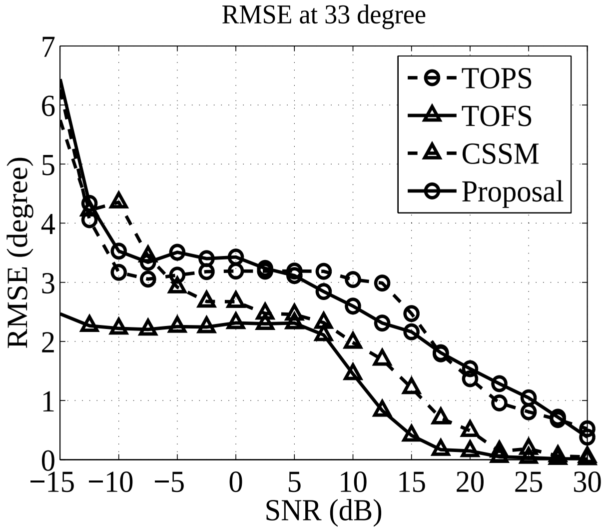

Fig. 3. RMSE for signal at 33°

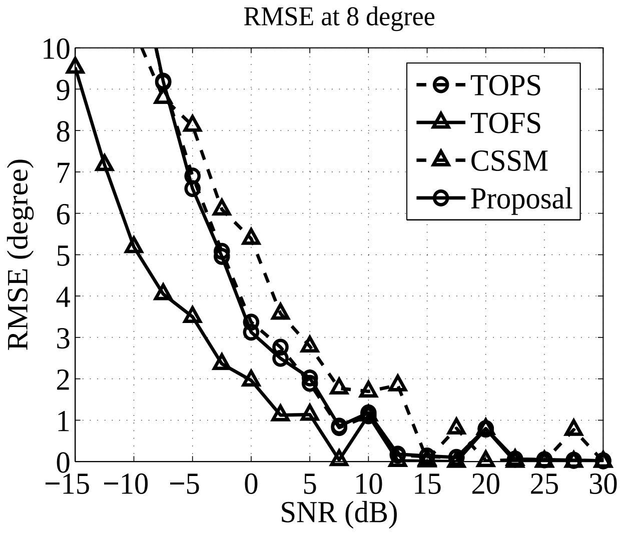

Fig. 2. RMSE for signal at 8°

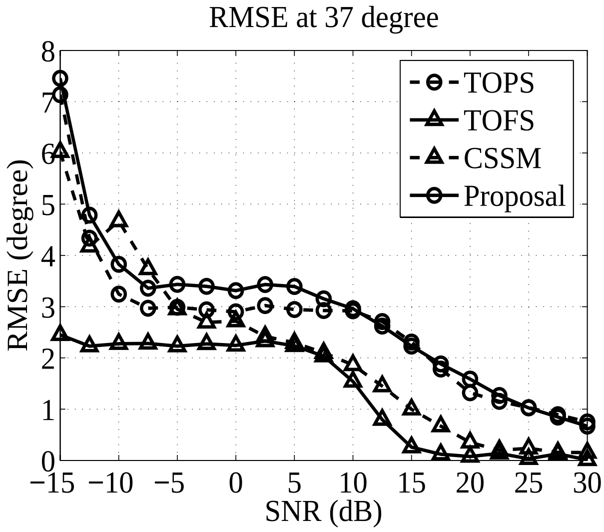

Fig. 4. RMSE for signal at 37°

noise subspaces. As a result, the row and column elements of the constructed submatrix correspond to each DOA. Therefore, the proposed method tests orthogonality of the signal subspace and the noise subspace in the row and column elements. We showed the performance of the resolution and the RMSE of the proposed method through computer simulations. In low SNR, it was shown that the resolution is improved compared to TOPS, because the proposed method correlates the signal subspace of each frequency by the frequency transformation. In mid SNR, it was shown that the estimation error of the proposed method is smaller than that of TOPS.

References

[1] H. Krim and M Viberg, "Two Decades of Array Signal Processing Research," *IEEE SIGNAL Process. Magazine*, vol. 13, no. 4, pp. 67–94, July 1996

[2] D. Lake, "Harmonic phase coupling for battlefield acoustic target identification," *Acoust., Speech and Signal Process.*, Vol. 4, pp. 2049–2052, May 1998.

[3] M. Wax, T. SHAN and T.Kailath, "Spatio-temporal spectral analysis by eigenstructure methods," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-32, no. 4, pp. 817–827, Aug. 1984.

[4] H. Hung and M. Kaveh, "Coherent wideband ESPRIT method for direction-of-arrival estimation of multiple wideband sources," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 38, no. 2, pp. 354–356, Feb. 1990.

[5] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. AP-34, no. 3, pp. 276–280, Mar. 1986

[6] H.Wang and M.Kaveh, "Coherent signal-subspace processing for the detection and estimation of angles of arrival of multiple wideband sources," *IEEE Trans. Acoust., Speech, Signal Proc.*, vol. ASSP-33, no. 4, pp. 823–831, Aug. 1985.

[7] E. Di Claudio and R. Parisi, "WAVES: Weighted average of signal subspaces for robust wideband direction finding," *IEEE Trans. Signal Proc.*, vol. 49, no. 10, pp. 2179–2190, Oct. 2001.

[8] Y. Yoon and L. M. Kaplan, “TOPS : New DOA Estimator for Wideband Signals, ”*IEEE Trans. on Signal Proc.*, vol. 54, no. 6, pp. 1977–1989, June 2006.

[9] H. Yu and J. L. Huang, “A new method for wideband DOA estimation,”*Wireless Communications, Networking and Mobile Computing, 2007. WiCom 2007.*, pp. 598–601, Sept. 2007.

[10] M. Doron and A.Weiss, “On focusing matrices for wideband array processing,” *IEEE Trans. Signal Proc.*, vol. 40, no. 6, pp. 1295–1302, June 1992.