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Fast STF Model and Applications on EEG Analysis

Yodchanan Wongsawat* and Soontorn Oraintara†
* Department of Biomedical Engineering, Mahidol University, Salaya, Nakornpathom 73170, Thailand
E-mail: egyws@mahidol.ac.th
† Department of Electrical Engineering, University of Texas at Arlington, TX 76019, USA
E-mail: oraintar@uta.edu

***Abstract*—Searching for the tool that can efficiently summarize a multi-channel EEG signal is a challenging problem in EEG processing. In this paper, we propose the fast implementation of the 3-way parallel factor analysis (PARAFAC) called Fast STF model (fSTF model) which can simultaneously employ all the space, time, and frequency domains of a multi-channel EEG. The multi-channel EEG signal is first subdivided along space and time domains into the selected numbers of segments. By carefully selecting the number of segments according to the structure of the brain, signatures (features) extracted from the fSTF model are comparable with those from the conventional STF model while the time used in computation is reduced by more than 50%. Signatures obtained from the fSTF model are further summarized as a single number to indicate the quality of the multi-channel EEG signal. The simulation results illustrate the merits of the proposed model via the applications on eyeblink artifact-contaminated EEG decomposition and EEG quality assessment.**

I. Introduction

Multi-channel electroencephalogram (EEG) is widely known for its potential in real-time brain understanding. In order to understand this multi-channel signal, all information should be incorporated to form the model. Therefore, finding the right model to extract the features of this signal with less time consuming becomes one of the challenging problems in biomedical engineering and neuroscience.

EEG is first modeled by its frequency statistics in [1]. The model is further improved by using time-frequency representation of a single channel EEG, [2], [3], [4] which is known as a nonstationary signal. Usually, EEG signals are recorded at multiple locations, yielding information about which part of the brain is functioning. This spatial knowledge is efficiently exploited using principal component analysis (PCA) in [5], [6]. However, by using PCA nonuniqueness occurs due to arbitrary choice of rotational axes [11], which leads to the robustness problem of the model. Recently, independent component analysis (ICA) is applied to eliminate this nonuniqueness problem by imposing the statistical independent constraint which is even stronger than orthogonality of PCA, [7], [8]. In conventional PCA and ICA, no frequency knowledge is exploited even though it can be separately employed later. All space, time, and frequency domains are employed in [9] by analyzing the region of time-frequency plane. Another interesting work on topographic-time-frequency decomposition is proposed in [10] by imposing the minimum norm and maximal smoothness to the time and frequency signatures, respectively, for uniqueness of the model. Recently, Miwakeichi *et al* [11] found that by using parallel factor analysis (PARAFAC), [12], [13] these uniqueness constraints are unnecessary. Therefore, they propose a novel model that applies space-time-frequency representation of a multi-channel EEG to a 3-way PARAFAC to obtain the space, time and frequency signatures (features), called space-time-frequency model (STF model). Although, all domains are exploited in these models, they suffer from the high computational complexity when measured in a long period of time or with high number of electrodes.

Previously in [14], we have presented two methods which can reduce the computational complexity of the STF model for a multi-channel EEG. The first method aims to estimate the STF model using the space-time-frequency-time/segment model (STF-TS model) by sub-dividing the time domain into a number of segments resulting in a 4-D array signal. The 4-way PARAFAC is then applied for the analysis of the 4-D array signal. This approach is appropriate when the signals are recorded for a long period of time. The second method aims to estimate the STF model using the space-time-frequency-space/segment model (STF-SS model), which is suitable when the number of channels (dimension of space domain) is high. By partitioning the channels into sub-groups, a 4-D array signal is constructed, and the 4-way PARAFAC is then applied for the analysis. However, if the dimensions of both time and space domains are high, the computation of these models can be further reduced. Therefore, in this paper, we extend the concept of the previous works by simultaneously partitioning the multi-channel EEG in both space and time domains called Fast Space-Time-Frequency model (fSTF model). The proposed reduced complexity model is further shown to be useful in the application on eyeblink artifact-contaminated EEG analysis and multi-channel EEG quality assessment.

II. Space-time-frequency model

This section reviews some backgrounds on the STF model. Each channel of the 1-D time-domain signal is first transformed to reveal its 2-D time-frequency representation. By stacking all the 2-D time-frequency arrays from all the channels, we form a 3-D array in space-time-frequency domain. Then, the 3-way parallel factor analysis (PARAFAC) [12] is further applied to this 3-D array in order to decompose the data into its fundamental components yielding the STF model.

A. Time-frequency transform

In order to map a 1-D signal in time domain to a 2-D signal in time-frequency domain, time-frequency transform is employed. Time-frequency modeling is known to be practical for the analysis of 1-D nonstationary signals e.g. EEG [4], [17]. There are two main methods to achieve this goal, i.e. to simultaneously localize signals in both time and frequency domains, the Cohen's class (translate signal in time and frequency) and the affine class (translate signal for time resolution and scale the signal for frequency resolution). Since the affine class yields nonuniform nature of time-frequency signal components, it is more suitable for EEG [11]. An EEG signal, $s(t)$, can be efficiently decomposed into the affine class time-frequency atoms by convolving with the complex Morlet wavelet basis (filter), $w(f,t)$, as

$$\acute{y}(f,t) = |w(f,t) * s(t)|^2 . \tag{1}$$

By stacking $\acute{y}(f,t)$ of all channels, a 3-D array can be formulated as $\acute{y}(n,f,t)$, where n is the channel index [11].

B. Space-time-frequency model (STF model)

In order to decompose a 3-D array signal into space, time and frequency domains, the 3-way PARAFAC is applied to the 3-D array signal, $\acute{y}(n,f,t)$ (denoted in array form as $\underline{\acute{\mathbf{Y}}}$) resulting in the STF model, which can be formulated as

$$\underline{\acute{\mathbf{Y}}}_{\ N\times F\times T} = h(\acute{\mathbf{A}},\acute{\mathbf{C}},\acute{\mathbf{D}}) + \underline{\acute{\mathbf{E}}}_{\ N\times F\times T}, \tag{2}$$

where the 3-way PARAFAC model, i.e. the STF model, is

$$h(\acute{\mathbf{A}},\acute{\mathbf{C}},\acute{\mathbf{D}}) = \sum_{m=1}^{M} \acute{a}(n,m)\acute{c}(f,m)\acute{d}(t,m),$$

and $\underline{\acute{\mathbf{E}}}$ is a 3-D array residual of the model. Each column of $\acute{\mathbf{A}}_{N\times M}$ denotes a space signature of the m-th component where its matrix elements are denoted as $\acute{a}(n,m)$, n is the channel index ranging from 1 to N, m is the component index ranging from 1 to M, and M is the number of components. Each column of $\acute{\mathbf{C}}_{F\times M}$ denotes the frequency signature where its matrix elements are denoted as $\acute{c}(f,m)$ and f is the frequency index ranging from 1 to F. Each column of $\acute{\mathbf{D}}_{T\times M}$ denotes the time signature where its matrix elements are denoted as $\acute{d}(t,m)$, and t is the time index ranging from 1 to T. It is noted that a suggested number of components M should be the one that maximizes the core consistency diagnostic (CORCONDIA) value which in [12] is known as an efficient model validation criteria. The parameters $\acute{\mathbf{A}}$, $\acute{\mathbf{C}}$, and $\acute{\mathbf{D}}$ can be estimated by using the alternate least square algorithm (ALS) [12] where the cost function is

$$\text{argmin}_{\acute{a},\acute{c},\acute{d}} \left\| \underline{\acute{\mathbf{Y}}} - \sum_{m=1}^{M} \acute{a}(n,m)\acute{c}(f,m)\acute{d}(t,m) \right\| .$$

Intuitively, the space signatures in $\acute{\mathbf{A}}$ obtained from this STF model represent the weighting parameters of the inter-channel correlation among time-frequency representations of each channel. Taking into account that this STF model needs to simultaneously process a 3-D array signal, hence, if at least one of its three dimensions, i.e. space, time or frequency, is large, the decomposition will be very complex and makes this elegant model infamous for real-world applications.

III. Fast Space-Time-Frequency Model

This proposed model is motivated from our previous works in [14]. By segmenting the selected domains (space, time or both) the STF model with the additional domains called space/segment and time/segment can be obtained. In this section, we introduce the novel model which combines the merits of the previously proposed STF-TS and STF-SS models [14].

If a multi-channel EEG signal happens to have both high number of channels and long period of time, the STF-TS and STF-SS models might not be as useful as they are. Hence, in order to efficiently estimate the STF for this type of signal, the generalization of the STF-TS and STF-SS models called Fast Space-Time-Frequency model (fSTF model) are derived. First, the temporal domain of a multi-channel EEG signal is divided into segments yielding a 4-way array as the input data of the STF-TS model. After that all channels of the resulting 4-way array are equally divided into groups yielding a 5-way array, $y(s_n,s_t,n_3,t_3,f_3)$ (denoted in array form as $\underline{\mathbf{Y}}$), where s_n is the channel/segment index ranging from 1 to S_n, s_t is the time/segment index ranging from 1 to S_t, n_3 is the channel index ranging from 1 to N_3, f_3 is the frequency index ranging from 1 to F_3, and t_3 is the time index ranging from 1 to T_3. The 5-way PARAFAC is then applied to this 5-D array signal rendering the fSTF model. The fSTF model of the 5-D array $\underline{\mathbf{Y}}$ can be formulated by combining the time/segment and space segment/signatures together in one model, that is:

$$\underline{\mathbf{Y}}_{\ N_3\times S_t\times F_3\times T_3\times S_n} = f(\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D},\mathbf{G}) + \underline{\mathbf{E}}, \tag{3}$$

where the 5-way PARAFAC model, i.e. the fSTF model, $f(\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D},\mathbf{G})$, is equal to

$$\sum_{m=1}^{M} a(n_3,m)b(s_t,m)c(f_3,m)d(t_3,m), g(s_n,m),$$

and $\underline{\mathbf{E}}$ is now a 5-D array residual of the size $N_3\times S_t\times F_3\times T_3\times S_n$. Each column of $\mathbf{A}_{N_3\times M}$ denotes the space signature of the m-th component ranging from 1 to M where its matrix elements are denoted as $a(n_3,m)$. Each column of $\mathbf{B}_{S_t\times M}$ denotes the time/segment signature where its matrix elements are denoted as $b(s_t,m)$. Each column of $\mathbf{C}_{F_3\times M}$ denotes the frequency signature where its matrix elements are denoted as $c(f_3,m)$. Each column of $\mathbf{D}_{T_3\times M}$ denotes the time signature where its matrix elements are denoted as $d(t_3,m)$, and each column of $\mathbf{G}_{S_n\times M}$ denotes the space/segment signature where its matrix elements are denoted as $g(s_n,m)$. The parameters $\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}$, and $\mathbf{G}$ can be estimated by the ALS where the cost function is the $\text{argmin}_{a,b,c,d,g}$ of

$$\left\| \underline{\mathbf{Y}} - \sum_{m=1}^{M} a(n_3,m)b(s_t,m)c(f_3,m)d(t_3,m)g(s_n,m) \right\| .$$

It should be noted that T and N in the STF model are equal to $T_3 \times S_t$ and $N_3 \times S_n$ in the fSTF model, respectively.

IV. Estimation method for calculating the STF model from the fast STF model

In this section, we show that the reduced complexity STF models can be efficiently used for estimating the conventional STF model. Specifically, instead of directly calculating the space, time, and frequency signatures from the original data using the STF model as in section II, we can estimate these signatures by cascading the weighted versions of their local signatures obtained by the reduced complexity STF models.

To be precise, we aim to estimate the STF model from the fSTF model. According to (3), the time signatures of a multi-channel signal can be estimated by cascading all S_t segments of the time signatures $\mathbf{D}$ which are weighted by their corresponding time/segment signatures $\mathbf{B}$. Similarly, the space signatures of the multi-channels signal can be estimated by cascading all S_n segments of the space signatures $\mathbf{A}$ which are weighted by their corresponding space/segment signatures $\mathbf{G}$.

M can be selected according to the applications, e.g. $M = 1$ for EEG quality assessment, and $M = 2$ for EEG eyeblink artifact removal. S_t can be chosen so that each segment is smaller than some specific interested patterns, e.g. to explore the eyeblink artifact, each segment should be approximately 1.5 seconds long. Lastly, S_n can be selected according to the identifiable areas of the brain, i.e. frontal, parietal, occipital, and temporal. In this paper, for 24-channel EEG, we divide the space domain of a multi-channel EEG into 12 segments (2-channel for each segment) in order to capture the symmetry/asymmetry of the brain functional.

When the residual is neglected, the fSTF model can be written in a matrix form as

$$\underline{\mathbf{Y}}_{\;F_3 \times S_t \times S_n \times T_3 \times N_3} = \left(\mathbf{D}\Sigma_{\mathbf{B}_{s_t}}\right) \Sigma_{\mathbf{C}_{f_3}} \left(\mathbf{A}\mathbf{\Sigma}_{\mathbf{G}_{s_n}}\right)^T, \tag{4}$$

where $\Sigma_{\mathbf{C}_{f_3}}$ is the diagonal matrix with the f_3-th row of $\mathbf{C}$ along the diagonal, $\Sigma_{\mathbf{B}_{s_t}}$ is the diagonal matrix with the s_t-th row of $\mathbf{B}$ along the diagonal, and $\Sigma_{\mathbf{G}_{s_n}}$ is the diagonal matrix with the s_n-th row of $\mathbf{G}$ along the diagonal. $s_t = 1, ..., S_t$, $s_n = 1, ..., S_n$, and $f_3 = 1, ..., F_3$. The time signature $\acute{\mathbf{D}}$ of the STF model can be estimated from the fSTF model as

$$\acute{\mathbf{D}} \approx \left(\mathbf{D}\mathbf{\Sigma}_{\mathbf{B}_1}, \ldots, \mathbf{D}\mathbf{\Sigma}_{\mathbf{B}_{S_t}}\right)^T. \tag{5}$$

Similarly, the space signature $\acute{\mathbf{A}}$ of the STF model can be estimated from the fSTF model as

$$\acute{\mathbf{A}} \approx \left(\mathbf{A}\mathbf{\Sigma}_{\mathbf{G}_1}, \ldots, \mathbf{A}\mathbf{\Sigma}_{\mathbf{G}_{S_n}}\right)^T. \tag{6}$$

V. Parameter analysis

By decomposing the multi-channel EEG signal using the reduced complexity STF models, the number of free parameters [18], i.e. the number of elements that the PARAFAC needs to find, can be analyzed in Table I:

TABLE I
Parameter analysis of the STF model and the reduced complexity STF models

Models	Number of free parameters
STF	$P_{STF} = M(N + F + T)$
STF-TS	$P_{STF-TS} = M(N_1 + S_t + F_1 + T_1)$
STF-SS	$P_{STF-SS} = M(N_2 + F_2 + T_2 + S_n)$
fSTF	$P_{fSTF} = M(N_3 + S_t + F_3 + T_3 + S_n)$

- *STF-TS model*: Since T in the STF model is equal to $T_1 \times S_t$ in the STF-TS model, when T is large,
$$P_{STF-TS} << P_{STF}.$$
This means that less parameters need to be estimated and thus reduces the computational complexity of the PARAFAC algorithm.
- *STF-SS model*: Given that S_n is the number of segments in a space domain. N_2, F_2, and T_2 are the numbers of channels in one segment, the number of frequency index, and the number of time index, respectively. Since N in the STF model is equal to $N_2 \times S_n$ in the STF-SS model, when N is high,
$$P_{STF-SS} << P_{STF}.$$
- *fSTF model*: According to the STF-TS and STF-SS models, it is clear that when T and N are high,
$$P_{fSTF} << P_{STF-TS}, P_{STF-SS} << P_{STF}.$$

VI. Simulation results

The goal of this section is to investigate the performance of the fSTF model whether it is a good approximation of the STF model for the purpose of real-world applications. The usefulness of our proposed model will be demonstrated via two EEG analysis experiments, i.e. the decomposition of a multi-channel EEG contaminated by eyeblink artifacts and the application on EEG quality assessment.

A. Decomposition of a multi-channel EEG contaminated by eyeblink artifacts

In this experiment, we use a dataset of a 24-channel EEG signal (Fig.1). This signal is contaminated by approximately 2 Hz eyeblink artifacts in channels 3-10 at the time stems around 0.2, 2.8, 4.2, 7.2, and 8.9 seconds. The goal is to extract these eyeblink artifacts from the 24-channel EEG by using space, time, and frequency information. The conventional STF model and the proposed reduced complexity model is applied to this data.

1) Issue on the performance: In order to decompose the multi-channel EEG into the clean EEG and the artifact, the STF model with the number of components (M) equals two is selected. As mentioned in Section IV, $S_t = 18$ and $S_n = 12$ are selected.

The space signatures of the STF-TS, STF-SS, and fSTF models (Figs.2(d), (g), and (j), respectively) result in similar signatures with those obtained from the conventional STF model (Fig.2(a)). Intuitively, the first component of each model can efficiently extract eyeblink artifacts which mainly occur in channels 3-10. The time signatures of the STF model (Fig.2(b)) also contain similar information as the estimated time signatures derived from the STF-TS, STF-SS, and fSTF models (Figs.2(e), (h), and (k), respectively), i.e. the eyeblink artifacts can be distinguished from the background EEG. Even though segmenting the time domain as in the STF-TS and fSTF models (Fig.2(e) and (k)) can cause some distortions in time signatures, the peak locations which are corresponding to all five eyeblink artifacts occurring at times 0.2, 2.8, 4.2, 7.2, and 8.9 seconds can still be preserved. In this experiment, frequency of each eyeblink artifact is approximately 2 Hz. According to Figs.2(c), (f), (i), and (l), it is clear that the frequency component of the eyeblink can be well decomposed by the STF model and all of the reduced complexity models. The STF and STF-SS (Figs.2(c) and (i)) models give almost the same signatures, while there are some small distortions in those of the STF-TS and fSTF (Figs.2(f) and (l)) models. This is because segmenting the time domain would cause more effect on changing the fundamental frequency in some intervals than segmenting the space domain.

2) Issue on the complexity: According to section III, by using the STF model, we have to calculate the PARAFAC of the 3-way array $\underline{\acute{\mathbf{Y}}}_{N\times F\times T}$ of size $24\times 91\times 1800$. This process consumes a longer period of time due to the calculations of more free parameters compared with the STF-TS model in which $\underline{\hat{\mathbf{Y}}}_{N_1\times S_t\times F_1\times T_1}$ is of size $24\times 18\times 91\times 100$. The second and third rows of Table II illustrate the computational complexities of both the STF and STF-TS models in terms of the numbers of free parameters. By assuming that the computational complexity of the STF model is 1, the STF-TS model consumes only 0.121. It is noted that the free parameters can also be reduced by segmenting the space domain by using the STF-SS model. However, in this experiment, using the STF-SS model is not as efficient as using the STF-TS model since T_1 is much greater than N_2. Further improvement on reducing the computational complexity of the STF and STF-TS models can be done by using the fSTF model of the 5-way array $\underline{\mathbf{Y}}_{N_3\times S_t\times F_3\times T_3\times S_n}$ of size $2\times 18\times 91\times 100\times 12$. The fSTF model consumes 4% less complexity than the STF-TS model and 88.4% less than the STF model. The numbers of iterations used before the ALS converges in order to calculate the free parameters of all the models are also shown in the fourth row of Table II. The results imply that besides the efficiently approximated signatures as in Figs.2(d)-(l), all the proposed models also converge as quickly as the conventional STF model.

B. Application on EEG quality assessment

In order to efficiently evaluate the quality of EEG, we assume that high quality multi-channel EEGs will contain lower magnitude variations in time-frequency domain than the low quality ones. This means that since we have stacked the time-frequency plane in order to form the 3-D array, our total variation on the space signature can be used to evaluate the quality of the multi-channel EEG ,i.e.

$$\text{Score} = \frac{1}{S_n}\sum_{i=1}^{S_n}|\mathbf{A}_i - \mathbf{A}_{i-1}| \tag{7}$$

where, in this application on EEG quality assessment, the number of components M is set to one since we need to summarize all of the artifacts into one component. S_t and S_n of the fSTF model can be selected as in Section VI-A. Therefore, space signature, $\mathbf{A}$, is a vector where the element i-th in $\mathbf{A}$ can be denoted as $\mathbf{A}_i$.

According to Table III, the interval ranging from 2-second to 4-second which contains one eyeblink artifact results in the quality scores of 0.0434 and 0.0482 from the STF and the fSTF models, respectively. The interval ranging from 5-second to 7-second which contains clean EEG signals results in the quality scores of 0.0134 and 0.0180 from the STF and the fSTF models, respectively. Furthermore, the interval ranging from 0-second to 9-second which contains series of eyeblink artifacts results in the quality scores of 0.0477 and 0.0475 from the STF and the fSTF models, respectively. It is obvious that *the lower the scores we obtain, the better the quality of multi-channel EEGs we have*. We can see that the quality assessment scores of the STF model and the fSTF model are comparable,

TABLE II
FREE PARAMETERS AND NORMALIZED TIME COMPLEXITY CONSUMED BY THE STF AND STF-TS MODELS OF A LEFT EYEBLINK EEG SIGNAL (ASSUME THAT TIME CONSUMED BY THE STF MODEL= 1)

Models	STF	STF-TS	STF-SS	fSTF
Free parameters	3830	466	3810	446
Time complexity	1	0.121	0.994	0.116
No. of iteration	26	18	28	18

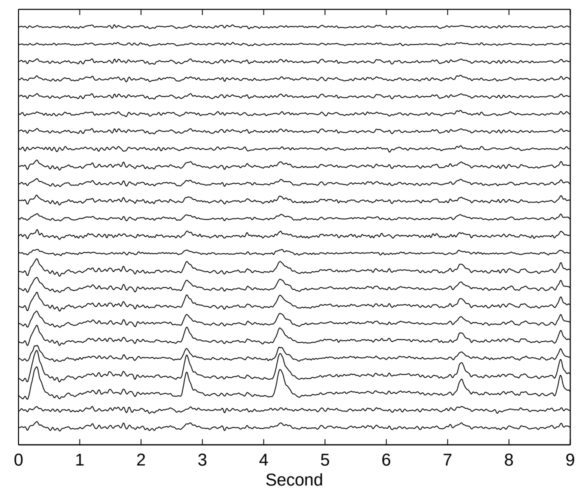

Fig. 1. Original 24-channel EEG contaminated by eyeblink artifacts (bottom to top lines named channels 1 to 24, respectively).

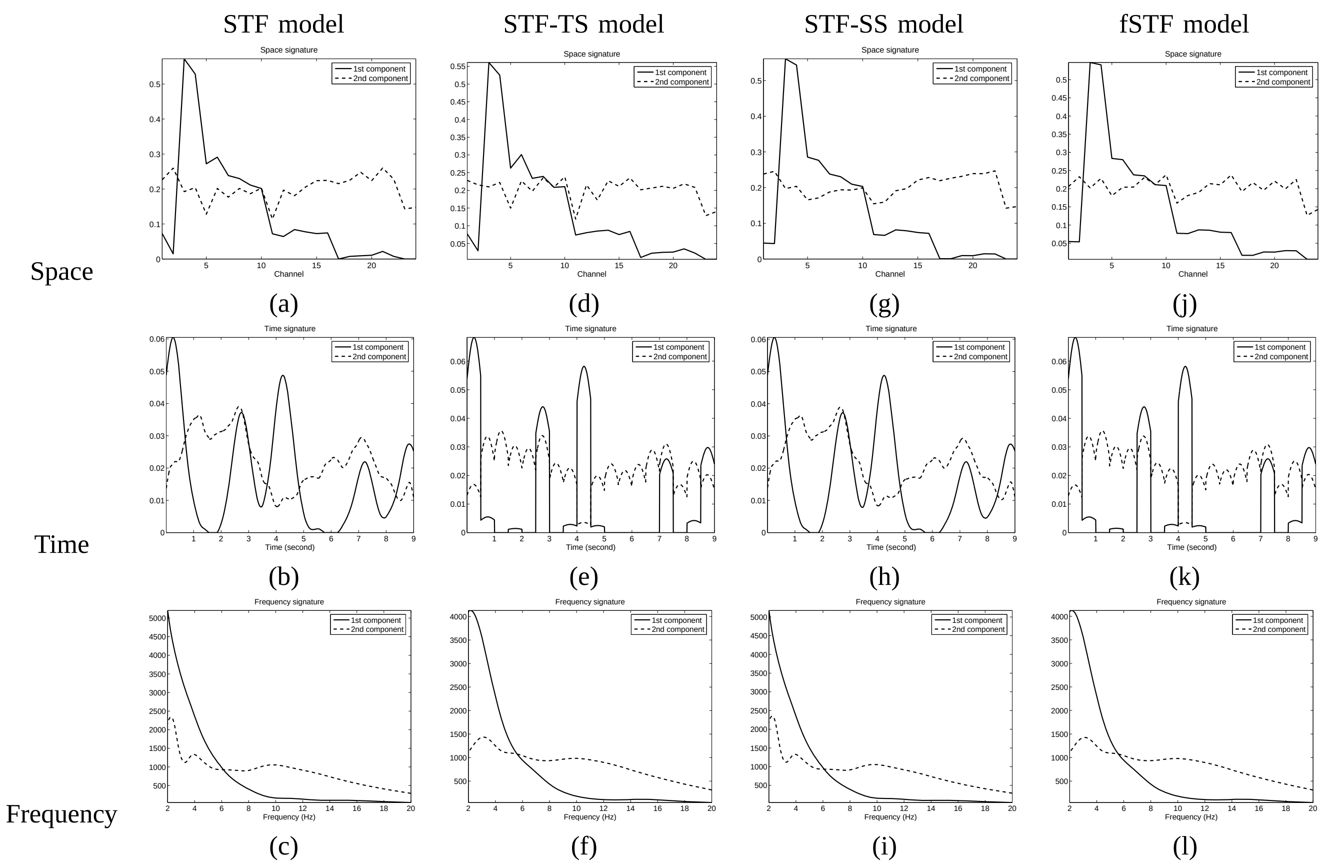

Fig. 2. Space signatures of the (a) STF, (d) STF-TS, (g) STF-SS, and (j) fSTF models. Time signatures of the (b) STF, (e) STF-TS, (h) STF-SS, and (k) fSTF models. Frequency signatures of the (c) STF, (f) STF-TS, (i) STF-SS and (l) fSTF models.

TABLE III
QUALITY ASSESSMENT SCORES OBTAINED FROM THE STF AND fSTF MODELS OF DIFFERENT MULTI-CHANNEL EEG SEGMENTS

Models	STF	fSTF
Quality Scores: Interval 2s to 4s	0.0434	0.0482
Quality Scores: Interval 5s to 7s	0.0134	0.0180
Quality Scores: Interval 0s to 9s	0.0477	0.0475

but it should be noted that computational complexity of the proposed model is dramatically reduced.

VII. Conclusions

We have presented a reduced complexity STF models named fSTF model. This proposed model can simultaneously exploit the space, time, frequency, space/segment and time/segment domains of a multi-channel EEG. We also derive the formulae for estimating the STF model from our proposed fSTF model. With less computational complexity, the proposed models can efficiently extract the eyeblink artifacts from the normal multi-channel EEG. Furthermore, the criterion for multi-channel EEG quality assessment has been proposed based on the STF model and the estimated space signature of the fSTF model. The proposed model yields comparable quality assessment score to the conventional STF model when being applied to the EEG contaminated by eyeblinks. According to the application on EEG quality assessment, various kinds of artifacts need to be investigated for our future work as mentioned in [22].

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