A Sequence Estimation Method based on EM Algorithm and its Application to the Watermark Detection Problem

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***Abstract*— In this paper, we address a sequence estimation problem formulated as a kind of blind deconvolution problem. We employ Bayesian estimation approach and treat this problem. As a solution to this problem, we present a sequence estimation method using EM (Expectation – Maximization) algorithm. The proposed method is applied to the watermark detection problem, which also becomes a blind deconvolution one. Numerical experiments using the DWT (Discrete Wavelet Transformation) based watermarking system show good performance as expected by us.**

I. Introduction

In our recent research concerning digital watermarking for images [1] [2], we have analyzed the watermark embedding and extracting processes and investigated how a watermark is distorted by attacks and image processing that wash the watermark away from a watermarked image. As a result, we have indicated that a watermark $\boldsymbol{w}$ is distorted in the matrix form of $\boldsymbol{u} = \boldsymbol{H}\boldsymbol{w}$ under some constraints and the watermark detection problem is to recover the watermark $\boldsymbol{w}$ by deconvolving the distorted one $\boldsymbol{u}$ with a matrix which reverses the effect of $\boldsymbol{H}$. The problem of this type appears in digital communication systems to estimate the transmitted data sequence $w(n)$ which is convolved with a filter having the impulse response $h(n)$ (the characteristic of a communication channel), giving a corrupted sequence $u(n) = \sum_k h(k)w(n-k)$. Some of the sequence estimation problems with the distortion model mentioned above are addressed and discussed along with a kind of blind deconvolution problem, in which no knowledge of $\boldsymbol{H}$ is assumed. This discussion focuses on a class of deconvolution algorithms based on the mutual – information maximization technique, the cumulant extrema technique, and other techniques with higher-order statistics [3]–[5].

In this paper, we treat this deconvolution problem using Bayesian estimation and, as a solution to this problem, we present an alternate algorithm for the sequence estimation using EM algorithm in the case that the distortion model is not known but its statistical characteristic like its mean and variance is evaluated or estimated. Further the proposed method is applied to the watermark detection problem. It is demonstrated from numerical experiments with the DWT-based watermarking system that the proposed method shows good improvement of the rate of watermark detection as wished by us.

II. Sequence Estimation using Bayesian Estimation

Let $\boldsymbol{w} = [w(n)]$ be an N-dimensional transmitted data sequence vector and let $\boldsymbol{u} = [u(n)]$ be the received one of N dimension. $\boldsymbol{u}$ is corrupted with an $N \times N$ matrix in the form of

$$\boldsymbol{u} = \boldsymbol{H}\boldsymbol{w} \tag{1}$$

The problem of sequence estimation is formulated as that of estimating $\boldsymbol{w}$ from an observation $\boldsymbol{r}$ of the distorted $\boldsymbol{u}$. In order to minimize the error of sequence estimation in consideration of the distortion model described by Eq.(1), we solve the problem with the following strategy. We first compute the probability density function (pdf) $p(\boldsymbol{s}|\boldsymbol{r})$ for a candidate $\boldsymbol{s}$ of $\boldsymbol{w}$ conditioned on $\boldsymbol{r}$, and then find $\boldsymbol{s}$ that maximizes $p(\boldsymbol{s}|\boldsymbol{r})$. By Bayes formula:

$$p(\boldsymbol{s}|\boldsymbol{r}) = \frac{p(\boldsymbol{r}|\boldsymbol{s})p(\boldsymbol{s})}{p(\boldsymbol{r})} \tag{2}$$

the maximization of the conditional pdf $p(\boldsymbol{s}|\boldsymbol{r})$ with respect to $\boldsymbol{s}$ for given $\boldsymbol{r}$ is equivalent to that of $p(\boldsymbol{r}|\boldsymbol{s})p(\boldsymbol{s})$. If $p(\boldsymbol{s})$ is uniform, then the maximum likelihood estimation that maximizes the likelihood function $p(\boldsymbol{r}|\boldsymbol{s})$ or the log-likelihood function $\log p(\boldsymbol{r}|\boldsymbol{s})$ is used.

We first consider the case that the distortion model $\boldsymbol{H} = [h_{ij}]$ is known or estimated. If the pdf of $\boldsymbol{r}$ and $\boldsymbol{s}$ is the Gaussian pdf, then

$$\begin{aligned} p(\boldsymbol{r}|\boldsymbol{s}) &= p(\boldsymbol{r}|\boldsymbol{H}, \boldsymbol{s}) \\ &= \frac{1}{\sqrt{(\pi\sigma^2)^N}} \, e^{-\frac{1}{\sigma^2}(\boldsymbol{r}-\boldsymbol{H}\boldsymbol{s})^t(\boldsymbol{r}-\boldsymbol{H}\boldsymbol{s})} \end{aligned} \tag{3}$$

$$\log p(\boldsymbol{r}|\boldsymbol{s}) \doteq -\frac{1}{\sigma^2}(\boldsymbol{r}-\boldsymbol{H}\boldsymbol{s})^t(\boldsymbol{r}-\boldsymbol{H}\boldsymbol{s}) \tag{4}$$

is obtained and the estimation is carried out by maximizing the above equations with respect to $\boldsymbol{s}$ for given $\boldsymbol{r}$. Here the suffix t stands for the transposition of matrices and the symbol $\doteq$ denotes equivalent for optimization purposes. Therefore the estimation problem results in the deconvolution problem of finding $\boldsymbol{s}$ such that $\boldsymbol{H}\boldsymbol{s} = \boldsymbol{r}$ for given $\boldsymbol{r}$ and $\boldsymbol{H}$.

In the case that $\boldsymbol{H}$ is unknown, the deconvolution problem becomes a kind of blind one. In the following, we treat this blind deconvolution problem. We consider the case that the distortion model $\boldsymbol{H} = [h_{ij}]$ is unknown but statistics of $\boldsymbol{H}$, like mean and variance of $\boldsymbol{H}$, are evaluated, and present a solution to this problem using the EM (Expectation – Maximization) algorithm[6], which consists of two steps called E-step (Expectation Step) and M-step (Maximization Step) and repeats these steps until convergence.

III. Sequence Estimation Based on EM Algorithm

When the distortion model $\boldsymbol{H}$ is unknown, the dependence of $p(\boldsymbol{s}|\boldsymbol{r})$ on the random distortion model is

$$p(\boldsymbol{s}|\boldsymbol{r}) = E[p(\boldsymbol{s},\boldsymbol{H}|\boldsymbol{r})] = \int_{\boldsymbol{H}} p(\boldsymbol{s},\boldsymbol{H}|\boldsymbol{r})d\boldsymbol{H} \tag{5}$$

where

$$p(\boldsymbol{s},\boldsymbol{H}|\boldsymbol{r}) = \frac{p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s})p(\boldsymbol{s})}{p(\boldsymbol{r})} \tag{6}$$

As the observation data is only $\boldsymbol{r}$ and incomplete data, that is, $\boldsymbol{H}$ is the hidden data, we cannot directly find out $\boldsymbol{s}$ that maximizes $p(\boldsymbol{s},\boldsymbol{H}|\boldsymbol{r})$ or $p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s})$. Accordingly, we first represent $p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s})$ as

$$p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s}) = p(\boldsymbol{H}|\boldsymbol{r},\boldsymbol{s})p(\boldsymbol{r}|\boldsymbol{s}) \tag{7}$$

$$\log p(\boldsymbol{r}|\boldsymbol{s}) = \log p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s}) - \log p(\boldsymbol{H}|\boldsymbol{r},\boldsymbol{s}) \tag{8}$$

and then calculate the conditional expectation of Eq.(8) with respect to $\boldsymbol{H}$ conditioned on an observation $\boldsymbol{u} = \boldsymbol{r}$ and a sequence estimation $\boldsymbol{w} = \hat{\boldsymbol{s}}$.

$$\begin{aligned}\log p(\boldsymbol{r}|\boldsymbol{s}) &= E[\log p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s})|\boldsymbol{r},\hat{\boldsymbol{s}}] \\ &- E[\log p(\boldsymbol{H}|\boldsymbol{r},\boldsymbol{s})|\boldsymbol{r},\hat{\boldsymbol{s}}]\end{aligned} \tag{9}$$

When we define

$$L(\boldsymbol{s}) = \log p(\boldsymbol{r}|\boldsymbol{s}) \tag{10}$$

$$Q(\boldsymbol{s}|\hat{\boldsymbol{s}}) = E[\log p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s})|\boldsymbol{r},\hat{\boldsymbol{s}}] \tag{11}$$

$$R(\boldsymbol{s}|\hat{\boldsymbol{s}}) = E[\log p(\boldsymbol{H}|\boldsymbol{r},\boldsymbol{s})|\boldsymbol{r},\hat{\boldsymbol{s}}] \tag{12}$$

then Eq.(9) can be written as

$$L(\boldsymbol{s}) = Q(\boldsymbol{s}|\hat{\boldsymbol{s}}) - R(\boldsymbol{s}|\hat{\boldsymbol{s}}) \tag{13}$$

It is noted that we would like to find $\boldsymbol{s}$ to maximize the log-likelihood $L(\boldsymbol{s})$ for $\boldsymbol{r}$, but we do not have the complete data to compute $L(\boldsymbol{s})$. So instead, we maximize $Q(\boldsymbol{s}|\hat{\boldsymbol{s}}) - R(\boldsymbol{s}|\hat{\boldsymbol{s}})$ given the observation $\boldsymbol{r}$ and our current estimate $\hat{\boldsymbol{s}}$. Here if $Q(\boldsymbol{s}|\hat{\boldsymbol{s}}) > Q(\hat{\boldsymbol{s}}|\hat{\boldsymbol{s}})$, then $L(\boldsymbol{s}) > L(\hat{\boldsymbol{s}})$ because $R(\boldsymbol{s}|\hat{\boldsymbol{s}}) \leq R(\hat{\boldsymbol{s}}|\hat{\boldsymbol{s}})$ holds according to Jensen's inequality. This is the basic idea behind the EM algorithm, from which we have the following algorithm expressed in two steps.
(a) E-step : Compute

$$\begin{aligned}Q(\boldsymbol{s}|\boldsymbol{s}^{(k)}) &= E[\log p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s})|\boldsymbol{r},\boldsymbol{s}^{(k)}] \\ &= \int_{\boldsymbol{H}} [\log p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s})]p(\boldsymbol{H}|\boldsymbol{r},\boldsymbol{s}^{(k)})d\boldsymbol{H}\end{aligned} \tag{14}$$

for the observation $\boldsymbol{u} = \boldsymbol{r}$ and the sequence estimate $\boldsymbol{w} = \boldsymbol{s}^{(k)}$ at the k-th iteration.
(b) M-step : Let $\boldsymbol{s}^{(k+1)}$ be the value of $\boldsymbol{s}$ which maximize $Q(\boldsymbol{s}|\boldsymbol{s}^{(k)})$, that is, find

$$\boldsymbol{s}^{(k+1)} = \arg\max_{\boldsymbol{s}} Q(\boldsymbol{s}|\boldsymbol{s}^{(k)}) \tag{15}$$

The EM algorithm consists of choosing $\boldsymbol{s}^{(0)}$, then performing the E-step and the M-step successively until convergence. Convergence may be determined by examining when the parameters quit changing, *i.e.*, stop.

In order to present the sequence estimation algorithm, we compute $Q(\boldsymbol{s}|\boldsymbol{s}^{(k)})$ of Eq.(14). We suppose that the pdf of $\boldsymbol{r}$, $\boldsymbol{s}$, $\boldsymbol{s}^{(k)}$, and $\boldsymbol{H}$ is the Gaussian pdf and compute $Q(\boldsymbol{s}|\boldsymbol{s}^{(k)})$. Let the mean and the variance of the elements h_{ij}'s of $\boldsymbol{H}$ be $E[h_{ij}] = e_{ij}$ and $E[(h_{ij} - e_{ij})(h_{kl} - e_{kl})] = \sigma_{ij}^2$ ($i = k$ and $j = l$); $= 0$ (otherwise), respectively, and let the pdf of $\boldsymbol{H}$ be the Gaussian pdf

$$p(\boldsymbol{H}) = \frac{1}{\sqrt{\pi^{N^2}\prod_{i=1}^{N}|\boldsymbol{\Lambda_i}|}}\prod_{i=1}^{N} e^{-(\boldsymbol{h_i}-\boldsymbol{e_i})^t\boldsymbol{\Lambda_i}^{-1}(\boldsymbol{h_i}-\boldsymbol{e_i})} \tag{16}$$

in which $\boldsymbol{h_i} = [h_{ij}]$ (column vector), $\boldsymbol{e_i} = [e_{ij}]$ (column vector), and $\boldsymbol{\Lambda_i} = E[(\boldsymbol{h_i} - \boldsymbol{e_i})(\boldsymbol{h_i} - \boldsymbol{e_i})^t]$ (diagonal matrix).

Under the assumption that $\boldsymbol{H}$ and $\boldsymbol{s}$ are statistically independent mutually, the conditional pdf $p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s})$ can be written as

$$\begin{aligned}p(\boldsymbol{r},\boldsymbol{H}|\boldsymbol{s}) &= \frac{p(\boldsymbol{r},\boldsymbol{H},\boldsymbol{s})}{p(\boldsymbol{s})} = \frac{p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s})p(\boldsymbol{H},\boldsymbol{s})}{p(\boldsymbol{s})} \\ &= \frac{p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s})p(\boldsymbol{H})p(\boldsymbol{s})}{p(\boldsymbol{s})} = p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s})p(\boldsymbol{H})\end{aligned} \tag{17}$$

while the conditional pdf $p(\boldsymbol{H}|\boldsymbol{r},\boldsymbol{s})$ can be expressed as

$$\begin{aligned}p(\boldsymbol{H}|\boldsymbol{r},\boldsymbol{s}) &= \frac{p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s})p(\boldsymbol{H},\boldsymbol{s})}{p(\boldsymbol{r},\boldsymbol{s})} \\ &= \frac{p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s})p(\boldsymbol{H})p(\boldsymbol{s})}{p(\boldsymbol{r},\boldsymbol{s})} = \frac{p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s})p(\boldsymbol{H})}{p(\boldsymbol{r}|\boldsymbol{s})}\end{aligned} \tag{18}$$

From Eq.(17) and Eq.(18), $Q(\boldsymbol{s}|\boldsymbol{s}^{(k)})$ can be written as

$$\begin{aligned}Q(\boldsymbol{s}|\boldsymbol{s}^{(k)}) &\doteq \int_{\boldsymbol{H}} [\log p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s})]p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s}^{(k)})p(\boldsymbol{H})d\boldsymbol{H} \\ &+ \int_{\boldsymbol{H}} [\log p(\boldsymbol{H})]p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s}^{(k)})p(\boldsymbol{H})d\boldsymbol{H}\end{aligned} \tag{19}$$

As the second term of the right-hand side of Eq.(19) is independent of $\boldsymbol{s}$ and is eliminated in M-step, we have

$$Q(\boldsymbol{s}|\boldsymbol{s}^{(k)}) \doteq \int_{\boldsymbol{H}} [\log p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s})]p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s}^{(k)})p(\boldsymbol{H})d\boldsymbol{H} \tag{20}$$

Using Eq.(3) and Eq.(16), we obtain

$$\begin{aligned}p(\boldsymbol{r}|\boldsymbol{H},\boldsymbol{s})p(\boldsymbol{H}) &= \frac{1}{\sqrt{\pi^{N^2}\prod_{l=1}^{N}|\boldsymbol{\Delta_l}|}} \\ &\times \prod_{l=1}^{N} e^{-(\boldsymbol{h_l}-\boldsymbol{d_l})^t\boldsymbol{\Delta_l}^{-1}(\boldsymbol{h_l}-\boldsymbol{d_l})}\end{aligned} \tag{21}$$

where

$$\boldsymbol{d_l} = E[\boldsymbol{h_l}|\boldsymbol{r},\boldsymbol{s}] = \boldsymbol{\Delta_l}\left(\frac{r_l}{\sigma^2}\boldsymbol{s} + \boldsymbol{\Lambda_l}^{-1}\boldsymbol{e_l}\right) \tag{22}$$

$$\boldsymbol{\Delta_l} = \left(\frac{1}{\sigma^2}\boldsymbol{s}\boldsymbol{s}^t + \boldsymbol{\Lambda_l}^{-1}\right)^{-1} \tag{23}$$

Substituting Eq.(3) and Eq.(21) into Eq.(20) and calculating $Q(\boldsymbol{s}|\boldsymbol{s}^{(k)})$, we have

$$\begin{aligned} Q(\boldsymbol{s}|\boldsymbol{s}^{(k)}) \ &\doteq\ -\sum_{l=1}^{N} r_l^2 + 2\boldsymbol{s}^t \left(\sum_{l=1}^{N} r_l \boldsymbol{d}_l^{(\boldsymbol{k})}\right) \\ &-\ \boldsymbol{s}^t \left\{\sum_{l=1}^{N}\left(\boldsymbol{d}_l^{(\boldsymbol{k})}\boldsymbol{d}_l^{(\boldsymbol{k})^t} + \boldsymbol{\Delta_l}^{(k)}\right)\right\}\boldsymbol{s} \quad (24) \end{aligned}$$

where

$$\boldsymbol{d_l}^{(k)} = E[\boldsymbol{h_l}|\boldsymbol{r},\boldsymbol{s}^{(k)}] = \boldsymbol{\Delta_l}^{(k)}\left(\frac{r_l}{\sigma^2}\boldsymbol{s}^{(k)} + \boldsymbol{\Lambda_l}^{-1}\boldsymbol{e_l}\right) \quad (25)$$

$$\boldsymbol{\Delta_l}^{(k)} = \left(\frac{1}{\sigma^2}\boldsymbol{s}^{(k)}\boldsymbol{s}^{(k)t} + \boldsymbol{\Lambda_l}^{-1}\right)^{-1} \quad (26)$$

In Eq.(25), $\boldsymbol{d_l}^{(k)} = E[\boldsymbol{h_k}|\boldsymbol{r},\boldsymbol{s}^{(k)}]$ $(k = 1, 2, \cdots, N)$ stands for the distortion model estimated from $\{\boldsymbol{r}, \boldsymbol{s}^{(k)}\}$. From these equations, we can obtain the updated value $\boldsymbol{s}^{(k+1)}$ of $\boldsymbol{s}$ that maximizes $Q(\boldsymbol{s}|\boldsymbol{s}^{(k)})$ of Eq.(24) by $\partial Q(\boldsymbol{s}|\boldsymbol{s}^{(k)})/\partial \boldsymbol{s} = 0$ where

$$\begin{aligned} \frac{\partial Q(\boldsymbol{s}|\boldsymbol{s}^{(k)})}{\partial \boldsymbol{s}} \ &=\ -2\left\{\sum_{l=1}^{N}\left(\boldsymbol{d}_l^{(\boldsymbol{k})}\boldsymbol{d}_l^{(\boldsymbol{k})^t} + \boldsymbol{\Delta_l}^{(k)}\right)\right\}\boldsymbol{s} \\ &+\ 2\left(\sum_{l=1}^{N} r_l \boldsymbol{d}_l^{(\boldsymbol{k})}\right) \quad (27) \end{aligned}$$

From this, we have the procedure to estimate $\boldsymbol{s}$ from given $\boldsymbol{r}$ as follows.

(i) Set the parameters σ^2, $\boldsymbol{e_l}$, and $\boldsymbol{\Lambda_l}$. Put $k = 0$ and choose an initial $\boldsymbol{s} = \boldsymbol{s}^{(0)}$.

(ii) Compute $\boldsymbol{\Delta_l}^{(k)}$ and $\boldsymbol{d_l}^{(k)}$ using

$$\boldsymbol{\Delta_l}^{(k)} = \left(\frac{1}{\sigma^2}\boldsymbol{s}^{(k)}\boldsymbol{s}^{(k)t} + \boldsymbol{\Lambda_l}^{-1}\right)^{-1} \quad (28)$$

$$\boldsymbol{d_l}^{(k)} = \boldsymbol{\Delta_l}^{(k)}\left(\frac{r_l}{\sigma^2}\boldsymbol{s}^{(k)} + \boldsymbol{\Lambda_l}^{-1}\boldsymbol{e_l}\right) \quad (29)$$

(iii) Update the value of $\boldsymbol{s}$ using

$$\begin{aligned} \boldsymbol{s}^{(k+1)} \ &=\ \left\{\sum_{l=1}^{N}\left(\boldsymbol{d}_l^{(\boldsymbol{k})}\boldsymbol{d}_l^{(\boldsymbol{k})^t} + \boldsymbol{\Delta_l}^{(k)}\right)\right\}^{-1} \\ &\times\ \left(\sum_{l=1}^{N} r_l \boldsymbol{d}_l^{(\boldsymbol{k})}\right) \quad (30) \end{aligned}$$

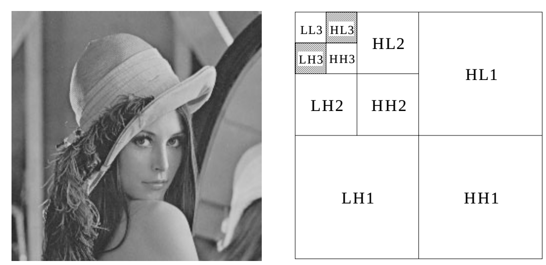
(iv) If $\|\boldsymbol{s}^{(k+1)} - \boldsymbol{s}^{(k)}\| \geq \epsilon$ for some $\epsilon(> 0)$, then set $k := k+1$ and go to step (ii), else put $\boldsymbol{s}^* = g(\boldsymbol{s}^{(k+1)})$ and decide that $\boldsymbol{s}^*$ is the tarnsmitted sequence. Here $g(\cdot)$ is an appropriate step function, by which we judge the sequence as a binary digit.

IV. Application of the Sequence Estimation Method to the Watermark Detection Problem and Numerical Experiments

In this section, we consider the DWT-based watermarking method [7] and apply the sequence estimation method to the watermark detection problem. Then, we carry out numerical experiments, in which we will investigate the robustness of watermark against image filtering, compression, etc., and verify the improvement of watermark detection using the proposed method. In this experiment, we use the test image LENNA with 256 $\times$ 256 pixels and 8 bit/pixel shown in Fig.1 (a).

The watermark embedding process is the following. The image LENNA is decomposed into ten subbands for three scales by using the Daubechies wavelet with 8-tap as shown in Fig.1 (b). L DWT coefficients $c(i_k)$ $(1 \leq k \leq L)$ with smaller magnitude are selected from the elements of the multiresolution representation components HL3 and LH3, and then $c(i_k)$ is modified according to a data bit $b(k)$ as $c(i_k) = Q$ for $b(k) = 1$ or $c(i_k) = -Q$ for $b(k) = 0$.

In this experiment, we prepare 4-dimensional watermark vector $\boldsymbol{w} = [w(k)]$ $(1 \leq k \leq 4)$ and make 5-dimensional code vector $\boldsymbol{v} = [v(l)]$ $(1 \leq l \leq 5)$ consisting of four data bits $v(l) = w(l)$ $(1 \leq l \leq 4)$ and one parity-check bit $v(5)$. (Hence we have 16 code vectors $\boldsymbol{v_i}$ $(1 \leq i \leq 16)$.) Then by putting $L = 25$ and $Q = 10$, one code vector $\boldsymbol{v}$ is embedded into LENNA five times, that is, $b(5k + l) = v(l)$ $(0 \leq k \leq 4, 1 \leq l \leq 5)$. The watermarked image quality PSNR is 53.7 [dB].



(a) (b)

Fig. 1. (a) Test image LENNA. (b) Wavelet decomposition of image.

In the watermark extracting process, L DWT coefficients $c'(i_k)$ $(1 \leq k \leq L)$ are extracted from the watermarked and attacked LENNA and four data bits $v(l)$, *i.e.*, watermark $w(l)$ $(1 \leq l \leq 4)$ is detected from $c'(i_k)$'s as follows.

(I) The conventional watermark detection method [7]

Checking the sign of $c'(i_k)$'s, we set $b'(k) = 1$ for $c'(i_k) \geq 0$ or $b'(k) = 0$ for $c'(i_k) < 0$. Next, for all l, we determine four data bits $v'(l)$ from $\{b'(5k+l), 0 \leq k \leq 4\}$ with decision by majority. If there is no error in $v'(l)$'s by parity-check, we decide that $v'(l)$ $(1 \leq l \leq 4)$ are embedded into LENNA, else we proceed to the next step of estimating data bits embedded into LENNA.

(II) The proposed watermark detection method

We compute the average $r(l)$ of $\{c'(i_{5k+l}) + Q\}/2Q$, $(0 \leq k \leq 4)$ for all l, i.e.,

$$r(l) = \frac{1}{10Q}\sum_{k=0}^{4}\{c'(i_{5k+l}) + Q\} \quad (0 \leq k \leq 4) \quad (31)$$

and set the observation data as $\boldsymbol{r} = [\ r(1)\ r(2)\ r(3)\ r(4)\ ]^t$. Next we estimate the embedded data bits from $\boldsymbol{r}$ using the proposed method with setting $N = 4$, in which the initial setting of parareters is as follows. Putting $\boldsymbol{\Lambda_i} = \sigma_\Lambda^2 \boldsymbol{I}$ $(i = 1, 2, \cdots, N)$ and $\sigma^2 = \sigma_\Lambda^2$, we set the value of σ as $\sigma = \sigma^{(k)} = \|\boldsymbol{r} - \boldsymbol{s}^{(k)}\|/N$ using the observation $\boldsymbol{r}$ and the k-th estimate $\boldsymbol{s}^{(k)}$, that is, σ is updated in the iteration. The setting method is determined by the advance experiment. Since degradation of the watermarked image by attacks and image processing is small, $\boldsymbol{e_i} = [e_{ij}]$ is set as $e_{ii} = 1$ and $e_{ij} = 0$ for $i \neq j$, that is, $\boldsymbol{E} = E[\boldsymbol{H}] = \boldsymbol{I}$, $\boldsymbol{I}$ being the identity matrix. The initial estimate of $\boldsymbol{s}$ is $\boldsymbol{s}^{(0)} = [\ v'(1)\ v'(2)\ v'(3)\ v'(4)\ ]^t$, whose elements are $v'(l)$'s obtained in step (I).

We first examine the robustness of the above watermarking system against smoothing, adding noise, and JPEG compression, in which watermark embedding and extracting are carried out for each of 16 code words $\boldsymbol{v_i}$'s.

We investigate the robustness by smoothing the watermarked LENNA with the mean filter

$$z(i,j) = \sum_{l,m=-1}^{1} \lambda_{l,m} y(i-l, j-m) \qquad (32)$$

where $\lambda_{l,m} = \eta$ for $(l,m) = (0,0)$; $= (1-\eta)/8$ for $(l,m) \neq (0,0)$ and $\eta = 0.1$. The smoothed image quality PSNR is 29.8[dB]. Figure 2 (a) shows the smoothed image, of which PSNR is 29.8[dB].

In the case of smoothing, we could detect all of data bits perfectly with the conventional method and did not need the proposed method.

Fig. 2. Watermarked images degraded and distorted through (a) smoothing, (b) adding noise, (c) JPEG compression, and (d) geometric distortion (StirMark)

We evaluate the robustness by adding Gaussian noise with the zero mean and the variance $\sigma^2 = 100$ to the watermarked LENNA. PSNR of the noisy image is 28.1[dB]. Figure 2 (b) illustrates the noisy image, whose PSNR is 28.1[dB].

In the case of adding noise, we could not detect 4 code words out of 16 ones with the conventional method. But applying the proposed method to these 4 code words, we could estimate the data bits correctly.

We test the robustness by compressing the watermarked LENNA using JPEG with parameter of 30 $\%$ quality. The compression image quality PSNR is 31.2[dB]. Figure 2 (c) shows the compression image, of which PSNR is 31.2[dB].

In the case of JPEG compression, we needed the estimation using the proposed method for half of 16 code words. As a result, the data bits could be estimated correctly, too.

We next examine the robustness of the above watermarking system against random geometric distortion in StirMark Attack [8]. The average PSNR of attacked images is 25.7 [dB]. The attacked image, whose PSNR is 25.7 [dB], is shown in Figure 2 (d). As is well known, this attack is strong and destroys almost all of data bits. Hence the conventional method ends in failure. So we prepare 20 observation data for one code word, that is, 320 ones in total, which cause detection error. Then we estimate the embedded data bits using the proposed method for each of 320 observation data. As a result, we could estimate the data bits for 310 observation data correctly, i.e., with accuracy of 97 $\%$.

V. Conclusion

In this paper, we have indicated that the sequence estimation problem is formulated as a kind of blind deconvolution problem and proposed the sequence estimation method using Bayesian estimation. Then we have considered the case that the distortion model of sequence is unknown but its statistics like mean and variance is evaluated or estimated, and presented the sequence estimation and decision algorithm using the EM algorithm. It is shown from numerical experiments with the DWT-based watermarking system that watermark can be detected with higher accuracy by using the proposed method in comparison with the conventional one.

The future work is to discuss the computational complexity of the proposed method, investigate the distortion model in digital communication, watermarking, etc., develop how to estimate its statistics, and so on. These related problems will be discussed in forthcoming papers.

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