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Differential Filters for the Identification of Dead Time

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Abstract In this paper, a generation method is proposed that is used for the identification of the dead time of a transfer function which parameters are known without the dead time. The proposed filters compose a group of several order differential FIR type digital filters. For non negative integer $r_{\max} \geq 0$, $r(0 \leq r \leq r_{\max})$ -th order differential filter is composed by cascade connection of r pieces of one order difference filter and $r_{\max} - r$ pieces of one order low pass filter and several order low pass filter. The amplitude values of them dividing with $(j\omega)^r$ are approximately same low pass characteristic. Moreover, these filters have ideal phase characteristics with several interval of the dead time. Using this filter group, we can show that the identification of the dead time of a transfer function is available. In this paper, the numerical examples of the proposed filters are presented.

1. Introduction

We consider a transfer function $G(s) = G_0(s)e^{-Ls}$ in which the dead time L is unknown and $G_0(s)$ is known. Where $G_0(s)$ is given by

$$G_0(s) = b(s)/a(s). \quad (1)$$

$$a(s) \equiv 1 + a_1 s + \cdots + a_n s^n \quad b(s) \equiv b_0 + b_1 s + \cdots + b_{n-1}s^{n-1}. \quad (2)$$

Let us define $x(t), y(t)$ for the input and output of $G(s)$ respectively. These situations are found in several instrumental environments for example the transfer time of the fluid in a long pipe, distance measurement by the ultra sonic sound. Almost all cases the correlation method is adopted for the identification of the dead time. For general case of unknown $G(s)$, several identification methods have been developed [1]. However, differential signals are too difficult to treat stably for the wide band noise. Therefore the methods which use some differential filters are very little. Author has been developed the identification methods using derivative digital filters. And derivative filters are designed in the cascade filter [2]-[6] or wavelet signal processing [7].

In order to estimate L we propose a method. The proposed method use several differential filters with transfer characteristics given by $H_\tau^{(r)}(s) = s^r H_0(s)e^{-\tau s}$ in which $H_0(s)$ is a low pass filter without any phase delay and is a real valued function. The signals $x_\tau^{(r)}(t)$ are outputs of $H_\tau^{(r)}(s)$ for $x(t)$, also $y_\sigma^{(r)}(t)$ are outputs of $H_\sigma^{(r)}(s)$ for $y(t)$. The configuration of these signals is shown in Fig.1. In this situation, if $\tau = \sigma + L$ is satisfied, we can conclude that the following relation (2) is satisfied.

$$y_\sigma^{(0)}(t) + \cdots + a_n y_\sigma^{(n)}(t) = b_0 x_\tau^{(0)}(t) + \cdots + b_{n-1}x_\tau^{(n-1)}(t) \quad (3)$$

Let us set the filter parameters as $\sigma < \tau_1 < \tau_2 < \cdots < \tau_K$ and $\tau_K > \sigma + L$. Also, let us define criterion functions by

$$E_k = \int_0^T e_k(t)^2 dt \quad (k = 1, 2, \ldots, K) \quad (4)$$

where T is a suitable large time, and $e_k(t)$ are the equation errors given by

$$e_k(t) = y_\sigma^{(0)}(t) + \cdots + a_n y_\sigma^{(n)}(t) - \{b_0 x_{\tau_k}^{(0)}(t) + \cdots + b_{n-1}x_{\tau_k}^{(n-1)}(t)\}. \quad (5)$$

The minimum of the sequences of $E_1, E_2, \ldots, E_K$ is attained by $k_{\min}$ that satisfies $E_{k_{\min}} = \underset{1 \leq k \leq K}{Min} E_k$. For this $k_{\min}$,

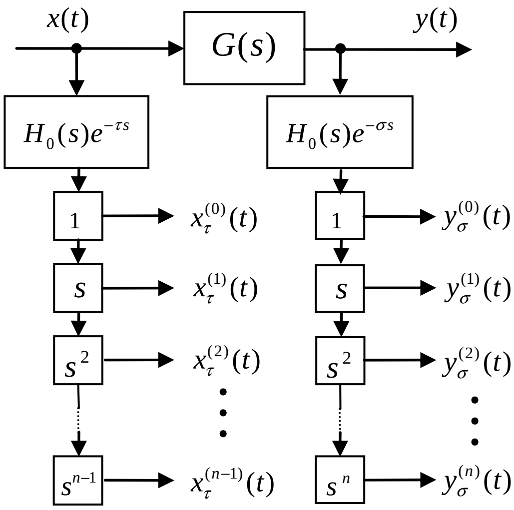

Fig.1 Differential Signals for Identification

we can estimate L as $L \approx \tau_{k_{\min}}$.

2. Evaluation of the Criterion Function

Let us evaluate the criterion function (4). Let $x(t)$ be a weak stationary stochastic process. By Kolmogorov's theorem it is shown [8]

$$x(t) = m_x + \int_{-\infty}^{\infty} e^{j\omega t} d\zeta(\omega). \quad (6)$$

Here, $m_x = E\{x(t)\}$ is the mean of $x(t)$, and $E\{\cdot\}$ shows the mean operation. And $\zeta(\omega)$ is an orthogonal measure that satisfies the following relations.

$$\zeta(\Lambda) = \bar{\zeta}(-\Lambda) \qquad E\{\zeta(\Lambda)\} = 0 \quad (7)$$

Λ is arbitrary Bair's set on $(-\infty, \infty)$. The auto correlation function of $x(t)$ is shown by

$$E\{\bar{x}(\tau)x(\tau + t)\} = \int_{-\infty}^{\infty} e^{j\omega t} dS(\omega). \quad (8)$$

Here, $S(\omega)$ is the spectral measure of $x(t)$ that satisfies

$$dS(\omega) = E\{|d\zeta(\omega)|^2\}. \quad (9)$$

Using the spectral partition of $x(t)$, we can express $y(t)$ by

$$y(t) = m_y + \int_{-\infty}^{\infty} G(j\omega)e^{j\omega t} d\zeta(\omega). \quad (10)$$

The output signals $x_\tau^{(r)}(t)$ from the differential filters $H_\tau^{(r)}(s)$ are shown by

$$x_\tau^{(r)}(t) = m_\tau^{(r)} + \int_{-\infty}^{\infty} H_\tau^{(r)}(\omega)e^{j\omega t} d\zeta(\omega). \quad (11)$$

If $r = 0$, then $m_\tau^{(0)} = m_x$, and else if $r > 0$, then $m_\tau^{(r)} = 0$. Therefore we can obtain

$$\sum_{\ell=0}^{n-1} b_\ell x_\tau^{(\ell)}(t) = b_0 m_x + \int_{-\infty}^{\infty} b(j\omega)H_0(\omega)e^{j(\omega t - \tau)} d\zeta(\omega). \quad (12)$$

Also, we obtain

$$\sum_{\ell=0}^{n} a_\ell y_\sigma^{(\ell)}(t) = m_y + \int_{-\infty}^{\infty} a(j\omega)H_0(\omega)G(j\omega)e^{j(\omega t - \sigma)} d\zeta(\omega)$$
$$= m_y + \int_{-\infty}^{\infty} b(j\omega)H_0(\omega)e^{j(\omega t - L - \sigma)} d\zeta(\omega). \quad (13)$$

Here $m_y = b_0 m_x$, we obtain

$$e_k(t) = \int_{-\infty}^{\infty} b(j\omega)H_0(\omega)(e^{-j\omega L} - e^{-j\omega(\tau_k - \sigma)})e^{j(\omega t - \sigma)} d\zeta(\omega). \quad (14)$$

If $\tau_{\hat{k}} = \sigma + L$, then obviously $e_{\hat{k}}(t) = 0$. By (14), $e_x(t)$ is a stationary process. Next, we consider the mean square of $e_x(t)$

$$E\{|e_k(t)|^2\} = \int_{-\infty}^{\infty} |b(j\omega)H_0(\omega)|^2 |1-e^{-j\omega(\tau_k-\sigma-L)}|^2 \, dS(\omega)$$

$$= 4\int_{-\infty}^{\infty} |b(j\omega)H_0(\omega)|^2 \sin^2\{\frac{\omega(\tau_k-\sigma-L)}{2}\} dS(\omega) \quad . \tag{15}$$

In the above discussion we assume that some noise exists in the output of the target system and we can use some ideal filter $H_\tau^{(r)}(s) = s^r H_0(s)e^{-\tau s}$. However, in some actual situation the output of the system contains noise $n_y(t)$, and $H_\tau^{(r)}(s)$ has some error $\Delta H_\tau^{(r)}$.

$$\tilde{H}_\tau^{(r)}(s) = s^r(H_0(s) + \Delta H_\tau^{(r)})e^{-\tau s} \tag{16}$$

We assume that the noise is a stationary process with the expression by

$$n_y(t) = \int_{-\infty}^{\infty} e^{j\omega t} d\zeta_n(\omega) \quad . \tag{17}$$

Here is spectral measure of $n_y(t)$. And $\zeta(\omega)$ and $\zeta_n(\omega)$ is not mutually correlated. Therefore, when we obtain $y_\sigma^{(r)}(t)$, we must replace $y(t) \to y(t) + n_y(t)$ and $e_x(t) \to \tilde{e}_x(t)$. By this replacement, we obtain new criterion function

$$\varepsilon_k = E\{|\tilde{e}_k(t)|^2\} = E\{|e_k(t)|^2\} + \Delta_e \quad . \tag{18}$$

$$\Delta_e \le \int_{-\infty}^{\infty} \{|a\Delta H_\tau^{(r)}|^2 + 4|bH_0\Delta H_\tau^{(r)}|^2\} dS(\omega)$$
$$+ \int_{-\infty}^{\infty} 4|a|^2 (|H_0|^2 + |\Delta H_\tau^{(r)}|^2) dS_n(\omega) \tag{19}$$

In this place, we assume the spectrum of the input for the target system is finite $S(\omega) = 0(|\omega| \ge \omega_0)$. Then Δ_e is finite. Moreover, $E\{|e_k(t)|^2\}$ is given

$$\frac{\pi^2\omega_0^2(\tau_k-\sigma-L)^2}{16} \int_{-\omega_0}^{\omega_0} |b(j\omega)H_0(\omega)|^2 \, dS(\omega) \le E\{|e_k(t)|^2\}$$
$$\le 4\sin^2\{\frac{\omega_0(\tau_k-\sigma-L)}{2}\} \int_{-\omega_0}^{\omega_0} |b(j\omega)H_0(\omega)|^2 \, dS(\omega). \tag{20}$$

Then the E_k is shown by the sum of the square of $\hat{L} = \tau_{\hat{k}} - \sigma$ and some constant. Therefore, we can estimate from $\tau_{\hat{k}}$ by the minimum of ε_k.

3. Generation of Differential Filters

Obviously, the differential filters $H_\tau^{(r)}(s) = s^r H_0(s)e^{-\tau s}$ are too difficult to generate in the analog techniques. Therefore, in this paper we try to approximate $H_\tau^{(r)}(s)$ by several digital filters. The sampling period T_S of the digital system is selected so as to satisfy that the folding frequency $f_F = 1/2T_S$ is far larger than the cutoff frequency of $H_0(s)$. We propose the following differential filters $D_m^{(r)}(z)$.

For the frequency f, if the relation

$$D_m^{(r)}(e^{j\omega}) \approx H_\tau^{(r)}(j2\pi f) \qquad (|f| \le f_F) \tag{21}$$

is satisfied, then we can use $D_m^{(r)}(z)$ for the substitute of $H_\tau^{(r)}(s)$, here $\omega = 2\pi f T_S$. This normalized frequency ω satisfies $|\omega| \le \pi$. The proposed digital differential filters are given by

$$D_m^{(r)}(z) = \{P(z)\}^r \{Q(z)\}^{r_{\max}-r} R_m(z) \tag{22}$$

where $0 \le r \le r_{\max}$ and

$$P(z) = 1 - z^{-1} \tag{23}$$
$$Q(z) = (1 + z^{-1})/2 \tag{24}$$
$$R_m(z) = \sum_{k=0}^{2m} \rho_m z^{-k} \quad . \tag{25}$$

The parameters $\rho_0, \rho_1, \ldots, \rho_{2m}$ of $R_m(z)$ satisfy $\rho_{2m-k} = \rho_k$ $(k = 0,1,\ldots,m)$. This means that $R_m(z)$ is the symmetric $2m$ order filter. For convention let us put $n = r_{\max} + 2m$ that is the order of $D_m^{(r)}(z)$. The frequency characteristics of $D_m^{(r)}(z)$ are shown by

$$D_m^{(r)}(e^{j\omega}) = j^r S_m^{(r)}(\omega) e^{-j(r\max/2+m)\omega} \tag{26}$$
$$S_m^{(r)}(\omega) = \omega^r U_m^{(r)}(\omega) \tag{27}$$
$$U_m^{(r)}(\omega) = \{P_0(\omega)\}^r \{Q_0(\omega)\}^{r_{\max}-r} T_m(\omega) \tag{28}$$
$$P_0(\omega) = \sin(\omega/2)/(\omega/2) \tag{29}$$
$$Q_0(\omega) = \cos(\omega/2) \tag{30}$$
$$T_m(\omega) = \rho_m + 2\sum_{k=0}^{m-1} \rho_k \cos\{(m-k)\omega\} \quad . \tag{31}$$

If we can set $m = [\tau/T_S - r_{\max}/2]$, then we can obtain $(r_{\max}/2 + m)\omega = (\tau/T_S)2\pi f T_S = 2\pi f \tau = s\tau$. Usually we can choose $\tau = vT_S\,(v:\text{integer})$, therefore this relation is easily satisfied. We choose $m_k = [\tau_k/T_S - r_{\max}/2]$ $(k = 1,2,\ldots,K)$ and generate $D_{m_k}^{(r)}(z)$ $(k = 1,2,\ldots,K)$. Moreover we add the condition that is shown by

$$U_{m_k}^{(r)}(\omega) \approx H_0(j2\pi f) \quad (k = 1,2,\ldots,K) \quad . \tag{32}$$

This important relation for the proposed filters shows that $U_{m_k}^{(r)}(\omega)$ of the proposed differential filters are approximately independent for the parameter $k\,(k = 1,2,\ldots,K)$ and $r = 0,1,\ldots,r_{\max}$. In order to approximate the relation, we select $\rho_0, \rho_1, \ldots, \rho_m$ by the minimum mean squared methods. The criterion functions are defined by

$$J_m = \int_0^\pi (T_m - H_0/P_0^r Q_0^{r_{\max}-r})^2 W(\omega) d\omega \quad . \tag{33}$$

Here $W(\omega)$ is a weighting function. The parameters $\rho_0, \rho_1, \ldots, \rho_m$ are determined by $\underset{\rho_0,\rho_1,\ldots,\rho_m}{Min} J_m$. Here, $H_0(j\omega)$ is a real valued function. In this paper, we define as

$$H_0(j\omega) = \begin{cases} 1 & 0 \le \omega \le \pi\rho_C \\ (\rho_R - \omega/\pi)/(\rho_R - \rho_C) & \pi\rho_C < \omega \le \pi\rho_R \\ 0 & \pi\rho_R \le \omega \le \pi \end{cases} \quad . \tag{34}$$

The weighting function is defined as

$$W(\omega) = \begin{cases} w_1 & 0 \le \omega \le \pi\rho_C \\ w_2 & \pi\rho_C < \omega \le \pi\rho_R \\ w_3 & \pi\rho_R \le \omega \le \pi \end{cases} \quad . \tag{35}$$

The values w_1, w_2, w_3 are non negative. The normal equation of (33) is given by

$$\mathbf{A\rho} = \mathbf{b} \tag{36}$$

$$\mathbf{A} \equiv \begin{bmatrix} a_{0,0} & \cdots & a_{0,m} \\ \vdots & \ddots & \vdots \\ a_{m,0} & \cdots & a_{m,m} \end{bmatrix} \quad \mathbf{b} \equiv \begin{bmatrix} b_0 \\ \vdots \\ b_m \end{bmatrix} \quad \mathbf{\rho} \equiv \begin{bmatrix} \rho_0 \\ \vdots \\ \rho_m \end{bmatrix} \quad . \tag{37}$$

The elements of $\mathbf{A}, \mathbf{b}$ are given for $k, \ell = 0,1,\ldots,m$ by

$$a_{k,\ell} = 2\sum_{k=0}^{m-1} \int_0^\pi \cos\{(m-k)\omega\} \cdot \cos\{(m-\ell)\omega\} W(\omega) d\omega$$

$$b_\ell = \int_0^\pi H_0 \cos\{(m-\ell)\omega\} W(\omega)/P_0^r Q_0^{r_{\max}-r} d\omega \quad . \tag{38}$$

The integration of $a_{k,\ell}, b_\ell$ is executed by numerical integration. For the algorithm of the integration we use Simpsons' formula. The partition number of interval of integral is set as n_{intg}. From the normal equation, we obtain the following result.

$$\mathbf{\rho} = \mathbf{A}^{-1}\mathbf{b} \tag{39}$$

Matrix inversion is executed by the Gauss' elimination method. Therefore we can obtain the solution for n within less than equal about 200. If we want to obtain the solution for larger value of n, we may use another method such as Gauss Seidel iterative method.

4. Examples of the proposed Filters

In order to generate the proposed filter, we constructed a program named as **CasLagFlt**. This was programmed by C# language included in MS Visual Studio 2008. The main form of this program is presented in Fig.2. When one push the [start] button, the dialog for parameter setting is opened shown in Fig.3. After one set the adequate parameters, he push the button [OK], then the design calculation begins and the frequency characteristics of $S_m^{(r)}(\omega), U_m^{(r)}(\omega), T_m^{(r)}(\omega)$ are depicted on the form.

For example, we set the following values for the parameters.

$$r_{\max} = 4, \rho_C = 0.2, \rho_R = 0.5, w_1 = 1, w_2 = 0, w_3 = 1, n_{\text{intg}} = 100 \quad (40)$$

In order check the characteristics $U_m^{(r)}(\omega)$ for several values of $n = 2m + r_{\max}$ which must have a constant shape, we calculate the frequency characteristics of $U_m^{(r)}(\omega)$ for $n = 20, 100, 200$. Fig.5, 6, 7 show $S_m^{(r)}(\omega)$, $U_m^{(r)}(\omega)$, $T_m^{(r)}(\omega)$ for $n = 20 (m = 8)$ respectively. Fig.8, 9, 10 also show $S_m^{(r)}(\omega)$, $U_m^{(r)}(\omega), T_m^{(r)}(\omega)$ for $n = 100$ respectively. Fig.11 shows $U_m^{(r)}(\omega)$ for $n = 200$. These examples show that the frequency characteristics of $U_m^{(r)}(\omega)$ are approximately equal. Therefore the proposed digital filter $D_m^{(r)}(z)$ for $n \geq 20$ is available to use the substitute of $H_\tau^{(r)}(s)$. Fig.12 shows $U_m^{(r)}(\omega)$ with another cut off parameter $\rho_C = 0.2$, $\rho_R = 0.22$ for which has sharp cut off property.

5. Example of Estimation

For example of $G(s)$ we consider the following system.

$$G(s) = \frac{e^{-0.2s}}{1+s} \quad (41)$$

And for the input signal to $G(s)$ we use

$$x(t) = \sin(2\pi f_1 t) + 0.2\sin(2\pi f_2 t). \quad (42)$$

Here $f_1 = 5[\text{Hz}]$, $f_2 = 13[\text{Hz}]$. We assume that the noise $n_y(t)$ is white and Gaussian with mean 0 and standard deviation 0.01. The sampling period of a digital system is $T_S = 0.02[\text{s}]$. The digital filters for the input and output of an objective system have the same parameters defined in (40) except n. For the input use we set $n_{In} = 120 \to 130$. And for the output use we set $n_{Out} = 100$. The evaluation time is set as $T = 1[\text{s}]$. On the above case, we obtain $\tau_k = (n_{In}/2)T_S$, $\sigma = (n_{Out}/2)T_S$. This means $n_{In} - n_{Out} = 20 \to 30$. That is $10T_S \leq L \leq 30T_S$. Therefore, we can estimate L in the region $0.1 \leq L \leq 0.3$. Fig.4 shows the calculated ε_k. By this figure, we can conclude that the estimated value of L is 0.2[s].

6. Conclusions

In this paper, a generation method is proposed that is used for the identification of the dead time of a transfer function. Some design examples of the proposed filters are presented. Also, an example of the identification for the dead time of a first order lag with dead time is presented. These results show that the proposed method is reasonable and available to estimate the dead time of a transfer function.

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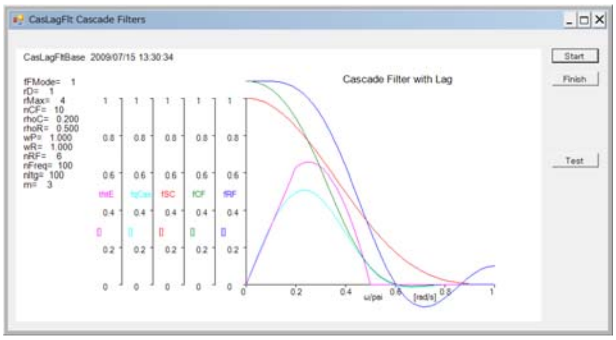

Fig.2 Main Form of Test Program

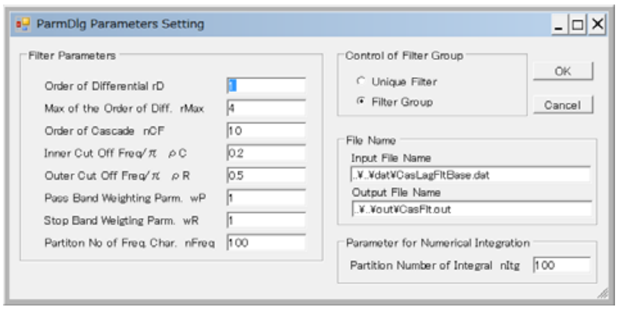

Fig.3 Dialog for Parameter Setting

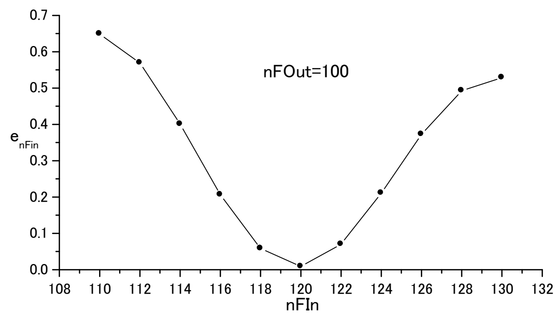

Fig.4 Calculated Estimation Value

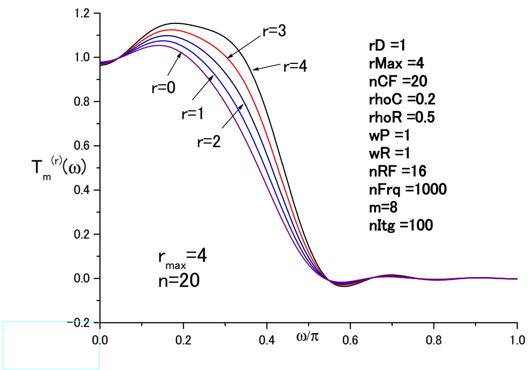

Fig.5 $T_m^{(r)}(\omega)$ *for* $n=20$

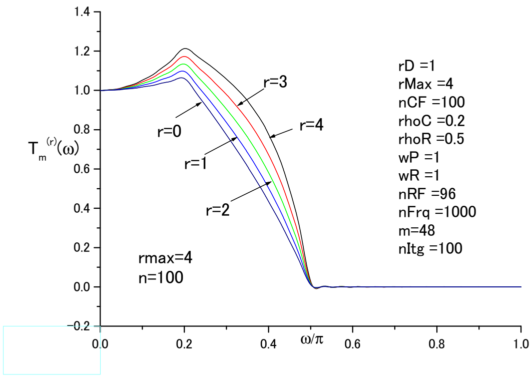

Fig.8 $T_m^{(r)}(\omega)$ *for* $n=100$

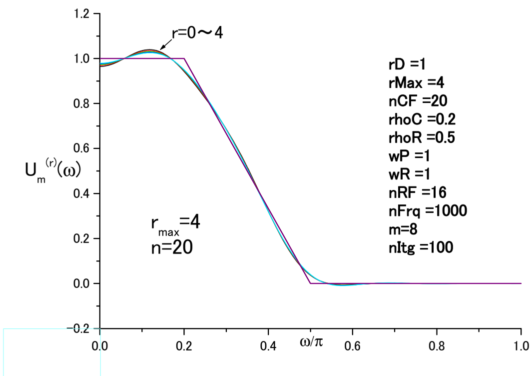

Fig.6 $U_m^{(r)}(\omega)$ *for* $n=20$

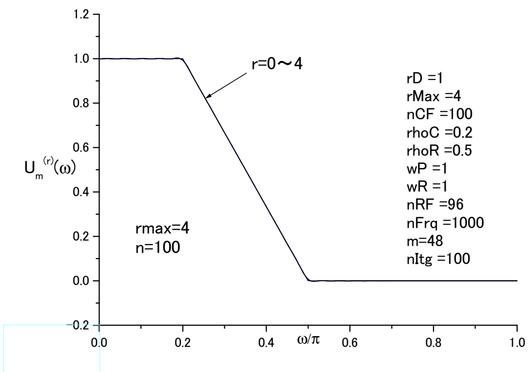

Fig.9 $U_m^{(r)}(\omega)$ *for* $n=100$

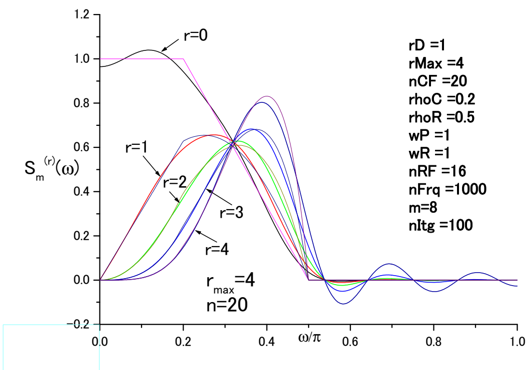

Fig.7 $S_m^{(r)}(\omega)$ *for* $n=20$

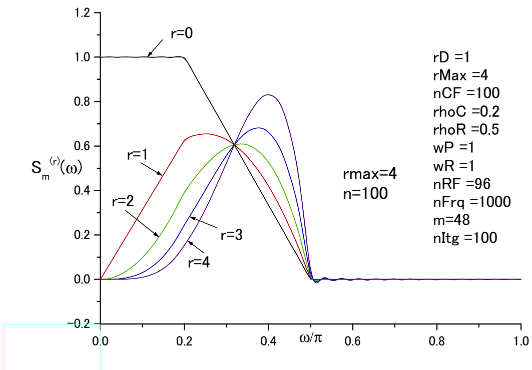

Fig.10 $S_m^{(r)}(\omega)$ *for* $n=100$

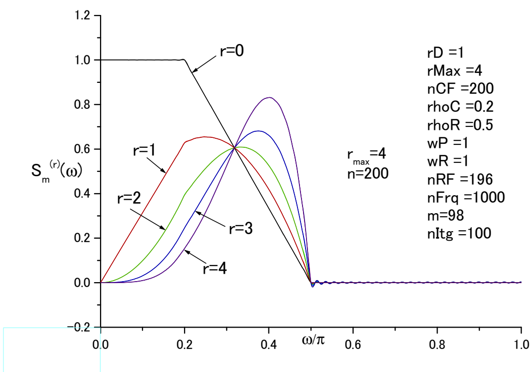

Fig.11 $S_m^{(r)}(\omega)$ *for* $n=200$

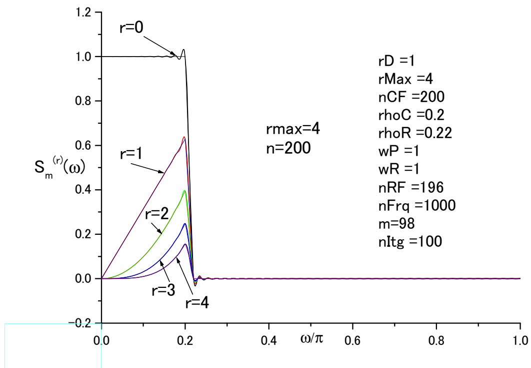

Fig.12 $T_m^{(r)}(\omega)\ n=200, \rho_C=0.2, \rho_R=0.22$