Estimation of Frequency Selective I/Q Imbalance and CFO for OFDM Systems

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***Abstract*—In this paper, we study the joint estimation and compensation of frequency selective in-phase and quadrature-phase (I/Q) imbalance and carrier frequency offset (CFO) for OFDM systems. We propose an iterative method to solve the joint estimation problem using training sequences. Unlike most earlier methods, we need only one arbitrary OFDM block for training. Simulation results show that the iterative algorithm converges very quickly and the proposed method has a good performance. The BER performance is only slightly worse than the ideal case where the I/Q mismatch, CFO and channel response are perfectly known at the receiver.**

I. INTRODUCTION

The application of direct-conversion receiver to orthogonal frequency division multiplexing (OFDM) systems has drawn a lot of attention in recent years. It is shown [1] that some drawbacks of direct-conversion receiver, such as in-phase and quadrature-phase (I/Q) imbalance and carrier frequency offset (CFO), can seriously degrade the performance of OFDM systems. The I/Q imbalance is due to the mismatches of the analog components between the I-branch and Q-branch. The mismatches include the amplitude and phase mismatches of the local oscillator, two mismatched low pass filters, amplifiers and analog-to-digital converters. The I/Q imbalance is in general frequency dependent. In [2], the authors show that the mismatches can be modeled as two equivalent filters and also propose some compensation methods. A method using one tap equalizer for the compensation of the frequency selective I/Q mismatch is proposed in [3]. In [4], the authors consider the joint estimation of the two mismatched filters and channel response using training sequence. Using repeated training sequences, an estimation method for CFO and I/Q imbalance is proposed in [5]. However an analytical solution of CFO is not provided in [5]. Thus a numerical search for CFO is needed. An improved method is proposed in [6], and moreover an analytical solution for CFO is also derived. An adaptive method for I/Q compensation is provided in [7]. Most of the above methods usually need more than one training block to achieve a good performance.

In this paper, we study the estimation and compensation of CFO and frequency selective I/Q mismatch for OFDM systems. Using training sequences, we propose an iterative method to solve the joint estimation problem. Unlike most earlier methods, we need only one arbitrary OFDM block for training. Simulation results show that the iterative algorithm converges very quickly and the proposed method has a good performance. The BER performance is almost indistinguishable from the ideal case where the I/Q mismatch, CFO and channel response are perfectly known at the receiver. The rest of the paper is organized as follows. The signal model of I/Q mismatch and CFO is given in Sec. 2. The proposed joint estimation method is described in Sec. 3. Simulation results are given in Sec. 4. A conclusion is drawn in Sec. 5.

II. SIGNAL MODEL

A communication system with a direct-conversion receiver is shown in Fig. 1 [5]. First assume that there is no CFO, i.e. $\Delta f = 0$. Its effect will be described later. In order to describe the baseband equivalent signal model, we define the two filters:

$$g_{R1}(t) \triangleq \frac{1}{2}(g_I(t) + \epsilon e^{-j\phi} g_Q(t)), \tag{1}$$

$$g_{R2}(t) \triangleq \frac{1}{2}(g_I(t) - \epsilon e^{-j\phi} g_Q(t)), \tag{2}$$

where ϵ is the amplitude mismatch, ϕ is the phase mismatch, and $g_I(t)$ and $g_Q(t)$ are the two low pass filters in the I- and Q-branch.

In this paper, we assume that the channel is a multipath channel with baseband representation

$$c(t) = \sum_{l=0}^{L_c} a_l e^{-j\theta_l} \delta(t - \tau_l), \tag{3}$$

where a_l is the path amplitude, θ_i is the path phase and τ_l is the path delay. Define the following two discrete-time equivalent channels

$$h_1(n) \triangleq \sum_{l=0}^{L_h} a_l e^{-j\theta_l} g_{R1}(nT - \tau_l), \tag{4}$$

$$h_2(n) \triangleq \sum_{l=0}^{L_h} a_l e^{-j\theta_l} g_{R2}(nT - \tau_l), \tag{5}$$

where T is the sample spacing and L_h is the maximum order of the two equivalent channels. It can be shown [5][7] that due to the I/Q imbalance, the distorted baseband received signal $r(n)$ is given by:

$$r(n) = h_1(n) \otimes x(n) + (h_2(n) \otimes x(n))^* + q(n), \tag{6}$$

where $\otimes$ denotes the linear convolution and $q(n)$ is the additive noise.

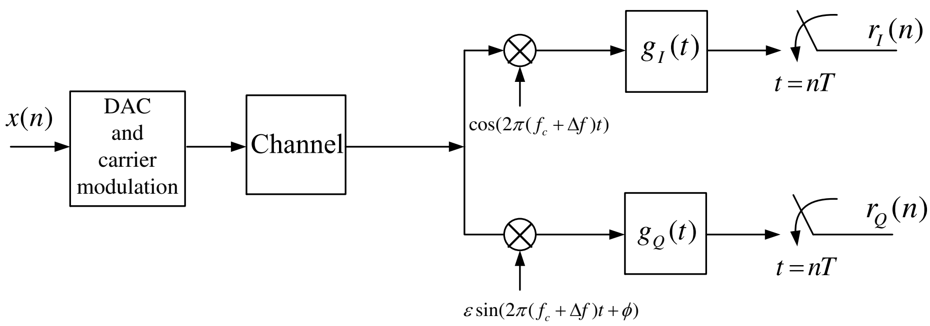

Fig. 1. A communication system with a direct-conversion receiver.

When there is no I/Q imbalance, that is $\epsilon = 1$, $\phi = 0$ and $g_I(t) = g_Q(t)$, we have $g_{R2}(t) = 0$ and $h_2(n) = 0$. Then the received signal in (6) reduces to

$$r(n) = h_1(n) \otimes x(n) + q(n), \tag{7}$$

which is the desired baseband signal. In practice, the I/Q mismatch is usually small. So it is in general true that the energy of $h_1(n)$ is much larger than that of $h_2(n)$, i.e.

$$\sum_{l=0}^{L_h} |h_1(l)|^2 \gg \sum_{l=0}^{L_h} |h_2(l)|^2. \tag{8}$$

Suppose in addition to I/Q imbalance, there is also CFO. Then the distorted baseband signal becomes [7]

$$\begin{aligned} r(n) &= \left(e^{j2\pi\Delta f nT} h_1(n) \otimes x(n)\right) \\ &+ \left(e^{j2\pi\Delta f nT} h_2(n) \otimes x(n)\right)^* + q(n). \end{aligned} \tag{9}$$

In this paper, we will show how to estimate the I/Q mismatch, CFO and channel response for OFDM transmissions. From [7], it is known that this problem is equivalent to the problem of estimating $h_1(n)$, $h_2(n)$ and Δf jointly.

III. PROPOSED ESTIMATION METHOD

In OFDM systems, the signals are transmitted in blocks. Let $\mathbf{s}$ be an OFDM block of length M consisting of modulation symbols such as QAM. After taking the M-point IDFT of $\mathbf{s}$ and a cyclic prefix (CP) of length L is appended, the sequence $x(n)$ is transmitted through the channel, as indicated in Fig. 1. In this paper, we assume that $L \geq L_h$. So there is no inter-block interference after removing the CP at the receiver. For notational simplicity, we assume $L = L_h$ in the following discussion. Let the normalized CFO $\Delta f MT$ be denoted by θ. Using the model in Sec. 2, the $M \times 1$ received baseband vector (with I/Q and CFO distortion) can be written as

$$\mathbf{r} = \mathbf{E}\mathbf{H}_{cir,1}\mathbf{x} + (\mathbf{E}\mathbf{H}_{cir,2}\mathbf{x})^* + \mathbf{q}, \tag{10}$$

where $\mathbf{E}$ is an $M \times M$ diagonal matrix

$$\mathbf{E} = \mathrm{diag}\left[\begin{array}{cccc} 1 & e^{j\frac{2\pi\theta}{M}} & \cdots & e^{j\frac{2\pi(M-1)\theta}{M}} \end{array}\right], \tag{11}$$

$\mathbf{H}_{cir,i}$ is an $M \times M$ circulant matrix with the first column

$$\mathbf{h}_i = \left[\begin{array}{cccccc} h_i(0) & \cdots & h_i(L) & 0 & \cdots & 0 \end{array}\right]^T, \tag{12}$$

$\mathbf{x}$ is the $M \times 1$ blocked version of $x(n)$, and $\mathbf{q}$ is the $M \times 1$ blocked version of $q(n)$. If θ and $h_i(n)$ are known, the I/Q imbalance and CFO can be compensated at the receiver. We can recover the transmitted vector $\mathbf{x}$ from the corrupted received vector $\mathbf{r}$. To see this, let us define

$$\mathbf{A} = \mathbf{E}\mathbf{H}_{cir,1}, \ \mathbf{B} = \mathbf{E}\mathbf{H}_{cir,2}. \tag{13}$$

Then (10) can be rewritten as

$$\mathbf{r} = \mathbf{A}\mathbf{x} + (\mathbf{B}\mathbf{x})^* + \mathbf{q}. \tag{14}$$

One can solve the above equation for $\mathbf{x}$, and it is given by

$$\widehat{\mathbf{x}} = \left(\mathbf{A} - \mathbf{B}^*\mathbf{A}^{-*}\mathbf{B}\right)^{-1}(\mathbf{r} - \mathbf{B}^*\mathbf{A}^{-*}\mathbf{r}^*). \tag{15}$$

Below we will show how to estimate θ, $h_1(n)$ and $h_2(n)$ when a training sequence is sent.

Suppose an OFDM input block is sent for training. That is, one $M \times 1$ vector $\mathbf{x}$, known to the receiver, is sent. For the purpose of estimation, we first rewrite the received vector $\mathbf{r}$ in (10) as

$$\mathbf{r} = \mathbf{E}\mathbf{X}_{cir}\mathbf{h}_1 + (\mathbf{E}\mathbf{X}_{cir}\mathbf{h}_2)^* + \mathbf{q}, \tag{16}$$

where $\mathbf{X}_{cir}$ is an $M \times M$ circulant matrix with the first column $\mathbf{x}$. Our goal is to jointly estimate $h_1(n)$, $h_2(n)$ and θ given the received vector $\mathbf{r}$. We first solve the two subproblems: (A) given $h_2(n)$, estimate $h_1(n)$ and θ and (B) given $h_1(n)$ and θ, estimate $h_2(n)$. Then the joint estimation of θ, $h_1(n)$ and $h_2(n)$ will be investigated.

(A) Given $h_2(n)$, Estimate $h_1(n)$ and θ: Assume that $h_2(n)$ is known. For a fixed θ, we substitute $h_2(n)$ and θ into (16). Using $\mathbf{E}^*\mathbf{E} = \mathbf{I}$, we obtain an estimate of $h_1(n)$ as

$$\widehat{\mathbf{h}}_1 = \mathbf{X}_{cir}^{-1}\mathbf{E}^*\left(\mathbf{r} - (\mathbf{E}\mathbf{X}_{cir}\mathbf{h}_2)^*\right). \tag{17}$$

The desired channel taps are the first $(L+1)$ entries of $\widehat{\mathbf{h}}_1$. The last $(M-L-1)$ entries of $\widehat{\mathbf{h}}_1$ are nonzero due to channel noise. For a fixed $h_2(n)$, any estimation error of θ will increase the energy of these entries. Exploiting this observation, we would like to find a θ that minimizes the energy of these entries. To do this, we first define an $(M - L - 1) \times M$ matrix $\mathbf{P}$ formed by the last $(M - L - 1)$ rows of the $M \times M$ identity matrix $\mathbf{I}_M$. Then we retain the last $(M - L - 1)$ entries of $\widehat{\mathbf{h}}_1$ as $\mathbf{P}\widehat{\mathbf{h}}_1$. Our problem is to find a θ that minimizes $\|\mathbf{P}\widehat{\mathbf{h}}_1\|^2$. Substituting (17) into $\mathbf{P}\widehat{\mathbf{h}}_1$, an estimate of CFO is given by

$$\widehat{\theta} = \arg\min_{\theta} \|\mathbf{P}\mathbf{X}_{cir}^{-1}\mathbf{E}^*(\mathbf{r} - (\mathbf{E}\mathbf{X}_{cir}\mathbf{h}_2)^*)\|^2. \tag{18}$$

After obtaining $\widehat{\theta}$, we obtain an estimate of $\mathbf{h}_1$ by substituting $\widehat{\theta}$ and $h_2(n)$ into (17). The desired $\widehat{h}_1(n)$ are the first $L+1$ entries of $\widehat{\mathbf{h}}_1$.

(B) Given θ and $h_1(n)$, Estimate $h_2(n)$: Suppose we know θ and $\mathbf{h}_1$. We can obtain an estimate of $h_2(n)$ by substituting θ and $\mathbf{h}_1$ into (16):

$$\widehat{\mathbf{h}}_2 = \mathbf{X}_{cir}^{-1}\mathbf{E}^*(\mathbf{r} - \mathbf{E}\mathbf{X}_{cir}\mathbf{h}_1)^*. \tag{19}$$

The desired $\widehat{h}_2(n)$ are the first $L+1$ entries of $\widehat{\mathbf{h}}_2$.

(C) An Iterative Algorithm for the Joint Estimation of $h_1(n)$, $h_2(n)$ and θ: Using the results in (A) and (B), we propose an iterative method to solve the joint estimation problem. The algorithm is described as follows:

1) Because $\|\mathbf{h}_2\|^2$ is small compared with $\|\mathbf{h}_1\|^2$, we can first assume that $\mathbf{h}_2 = 0$ and use Part (A) to get the initial estimates $\widehat{\theta}^0$ and $\widehat{\mathbf{h}}_1^0$. By setting $\mathbf{h}_2 = 0$ in (17) and (18), we get

$$\widehat{\theta}^0 = \arg\min_{\theta} \|\mathbf{P}\mathbf{X}_{cir}^{-1}\mathbf{E}^*\mathbf{r}\|^2 \tag{20}$$

$$\widehat{\mathbf{h}}_1^0 = \mathbf{X}_{cir}^{-1}(\widehat{\mathbf{E}}^0)^*\mathbf{r}. \tag{21}$$

2) Obtain an initial estimate $\widehat{\mathbf{h}}_2^0$ by substituting $\widehat{\mathbf{h}}_1^0$ and $\widehat{\theta}^0$ into (19).
3) At the $(i+1)$th iteration, we form the vector $\mathbf{E}\mathbf{X}_{cir}\mathbf{h}_2$ by substituting $\widehat{\theta}^i$ into $\mathbf{E}$ and substituting $\widehat{\mathbf{h}}_2^i$ into $\mathbf{h}_2$. Using (18), we can obtain a new estimate of CFO $\widehat{\theta}^{(i+1)}$ as

$$\widehat{\theta} = \arg\min_{\theta} \|\mathbf{P}\mathbf{X}_{cir}^{-1}\mathbf{E}^*(\mathbf{r} - (\widehat{\mathbf{E}}^i\mathbf{X}_{cir}\widehat{\mathbf{h}}_2^i)^*)\|^2. \tag{22}$$

4) Obtain $\widehat{\mathbf{h}}_1^{(i+1)}$ by substituting $\widehat{\theta}^{(i+1)}$ and $\widehat{\mathbf{h}}_2^i$ into (17).
5) Obtain $\widehat{\mathbf{h}}_2^{(i+1)}$ by substituting $\widehat{\theta}^{(i+1)}$ and $\widehat{\mathbf{h}}_1^{(i+1)}$ into (19).
6) Stop if the change in $\widehat{\theta}^{(i+1)}$, $\widehat{\mathbf{h}}_1^{(i+1)}$ and $\widehat{\mathbf{h}}_2^{(i+1)}$ is smaller than a given threshold such as 10^{-6} or the desired number of iterations is reached. Otherwise, $i = i+1$ and repeat steps 3-5.

IV. SIMULATION RESULTS

In this section, we carry out Monte-Carlo experiments to verify the proposed method. Both the training data and the information bearing data are QPSK. The size of the DFT matrix is $M = 128$. We assume that the multipath channel $c(t)$ in (3) has three paths. The path gains $a_l e^{-j\theta_l}$ are zero-mean i.i.d. complex Gaussian random variables with $\mathrm{E}\left\{a_l^2\right\} = (\frac{1}{2})^l$, and the corresponding path delays are $\tau_0 = 0, \tau_1 = 200ns$ and $\tau_2 = 400ns$. The normalized CFO is $\theta = -1.32$. The frequency independent I/Q mismatch parameters are assumed to be $\epsilon = 1.1$ and $\phi = 10°$. The frequency selective I/Q mismatch filters $g_I(t)$ and $g_Q(t)$ are modeled as Butterworth filters of order 7 with cutoff frequencies $8.8MHz$ and $9.24MHz$ respectively. So there is a mismatch between the two filters due to different cutoff frequencies. The sampling frequency at the receiver is $20MHz$. Using the above setup, the two equivalent discrete time channels $h_1(l)$ and $h_2(l)$ (with 98%

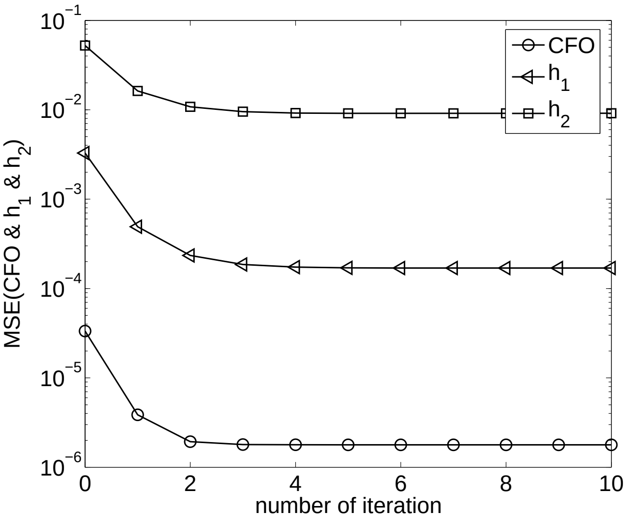

Fig. 2. MSE vs. iteration number

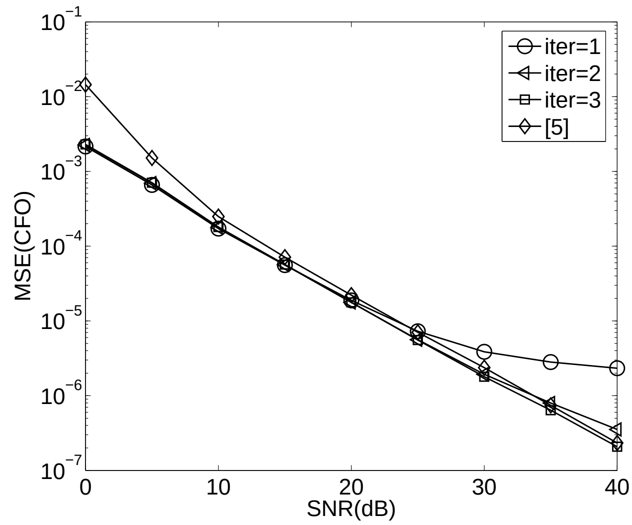

Fig. 3(a)

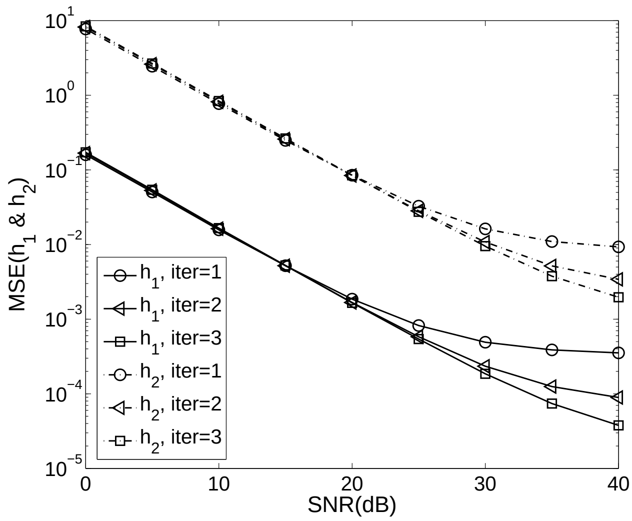

Fig. 3(b)

Fig. 3. MSE of (a) θ and (b) $\mathbf{h}_1$ and $\mathbf{h}_2$

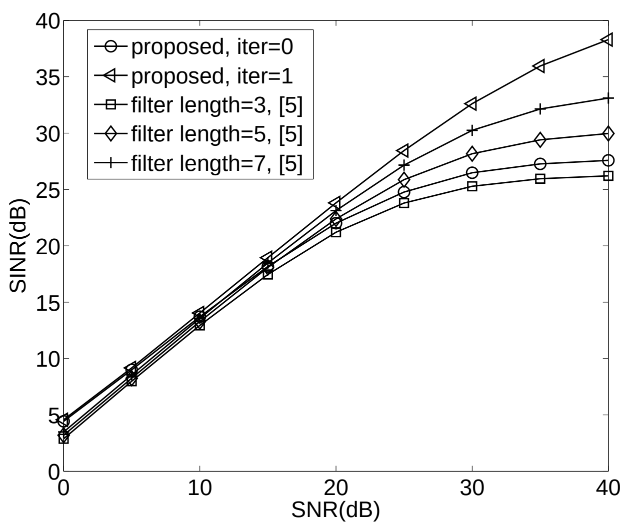

Fig. 4. SINR performance

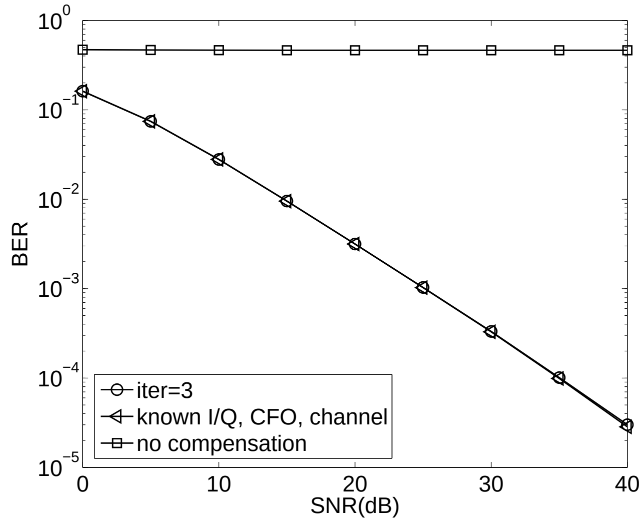

Fig. 5. BER performance

of energy) will have 17 taps. The CP length is 16 so that there is no inter-block interference.

Fig. 2 shows the MSE versus the number of iterations. The SNR is assumed to be $30dB$. From this figure, we find that increasing the number of iterations can improve performance. It is found that the proposed method converges quickly, and a good performance can be achieved after 3 iterations. Fig. 3 shows the MSE versus SNR for the estimation of θ, $h_1(n)$ and $h_2(n)$. For comparison, we draw the CFO performance of the method proposed in [5][1]. Notice that the method in [5] needs at least 3 repeated blocks to obtain a CFO estimate. Here we use 4 repeated blocks. The filter length for the I/Q compensation in [5] is assumed to be 5. Thus the CP length in [5] increases by 5. On the other hand, our proposed method needs only one OFDM block for training and also has shorter CP length. From Fig. 3(a), we see that for 3 iterations, the proposed method is comparable to [5] for high SNR and it is better than [5] for low SNR. For low SNR region ($\leq 25dB$), only 1 iteration is needed to give a satisfactory performance, whereas for high SNR region ($\geq 25dB$), the performance of 2 or 3 iterations is good enough.

Next we show signal-to-interference-noise ratio (SINR) and BER performance versus SNR. From Fig. 4, we see that increasing the order of the FIR filter can raise the output SINR for the method in [5]. Our proposed method can outperform the method in [5] significantly with only one iteration. In Fig. 5, we find that without proper compensation, the BER is poor. After 3 iterations, the system performance improves significantly. For comparison, we also plot the ideal case where the I/Q parameter, CFO and channel response are perfectly known at the receiver. From this figure, it is found that the BER of the proposed method is almost indistinguishable from the ideal case.

[1]Channel estimation is not considered in [5]

V. CONCLUDING REMARKS

In this paper, we study the joint estimation and compensation of frequency selective I/Q imbalance and CFO for OFDM systems. An iterative algorithm is proposed to solve this joint estimation problem. The proposed method needs only one arbitrary OFDM block for training. Simulation results show that the proposed iterative algorithm converges quickly and has a good performance. The BER is almost indistinguishable from the ideal case where the I/Q imbalance, CFO and channel response are perfectly known at the receiver.

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