A DIAGONALLY WEIGHTED SPACE-TIME BLOCK CODE OFDM WITH CHANNEL ESTIMATION

Shin-Yung Lin, Shih-Fu Hsieh and Ko-Chiang Li

Department of Communication Engineering, National Chiao Tung University,
Hsinchu, Taiwan 300, Republic of China
Tel: 886-3-5731974, E-mail: sfhsieh@mail.nctu.edu.tw

ABSTRACT

We focus on space-time block code (STBC) OFDM system with four transmit antennas. A block diagonal (BD) non-orthogonal STBC is proposed for full-rate transmission of complex data. To estimate the channel response, a BD STBC matrix has to be nonsingular for all possible transmitted BD STBC data matrices. A diagonal weighting constant is proposed to assure the nonsingularity. A phase-direct technique can be used to further improve the subspace-based semi-blind channel estimation. From computer simulation the proposed channel estimation is shown to be effective and we can also see that a large weighting constant improves channel estimation at the cost of a degraded bit error rate.

Index Terms **—** space-time code, OFDM, channel estimation

1. INTRODUCTION

Space-Time Block Code (STBC) Orthogonal Frequency Division Multiplexing (OFDM) has been popular in wireless communications for its advantages of transmit and time diversity to combat fading [1]. In case of four-transmit-antenna, non-orthogonal STBC [3] can achieve full transmission rate for complex data symbol, such as QPSK. Similar to previously proposed two non-orthogonal STBC's, a new block diagonal (BD) STBC is proposed and it can be shown to be the remaining one yet to be found.

At its receiver end, channel estimation is necessary for decoding and equalization. In case of a complex QPSK STBC OFDM system, the STBC data matrix can possibly be singular. As a result, its matrix inverse does not exist and the channel estimation fails. To overcome this singularity issue, we propose to impose a diagonal weighting constant on the STBC matrix.

A subspace-based semi-blind channel estimation has been proposed [4]. A phase-direct technique [5,6] can be employed to enhance its performance, but it only deals with the case of two-transmit-one-receive antenna and a real BPSK data. We will apply phase-direct channel estimation to the case of four-by-one antenna non-orthogonal STBC and QPSK data.

This paper is organized as follows. After presenting the STBC system model in section 2, we propose a diagonally weighted block diagonal STBC and the phase-direct channel estimation in section 3. Section 4 derives the performance analysis from which we can see effects of the weighting constant on channel estimation error and bit error rate. Section 5 shows our simulation results. Finally, our conclusions are summarized in section 6.

2. STBC MODEL

Fig. 1 shows a complete STBC OFDM transceiver in time domain. A block precoder is needed to apply the subspace-based channel estimation [4]. $\mathbf{W}_{IFFT}$ and $\mathbf{W}_{FFT}$ denote Fourier transform matrices, while $\mathbf{A}_{CP}$ and $\mathbf{R}_{CP}$ means the cyclic prefix insertion and deletion.

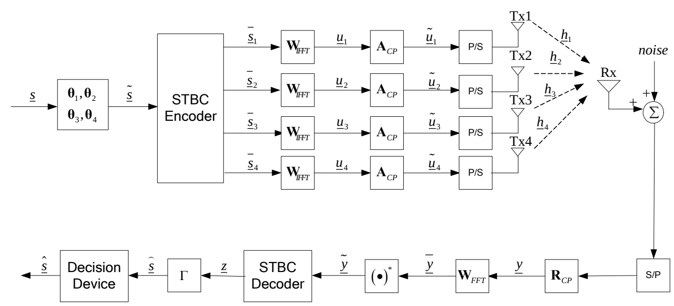

Fig. 1. Four-transmit-antenna STBC OFDM transceiver model with block precoders

For simplicity, a STBC transceiver system model in frequency domain is shown in Fig. 2. Suppose *4* transmit antennas are used. $\mathbf{S}$ denotes the STBC transmission matrix, made up by *4* symbol vectors s_1, s_2, s_3, s_4 and their conjugates. The channel frequency response vector and the AWGN are denoted by $\underline{\mathbf{h}}$ and $\underline{\mathbf{n}}$, respectively.
First, The ST encoder takes OFDM symbols to compose the transmission matrix $\mathbf{S}$, which is then fed into the channel. The received data vector $\underline{\mathbf{y}}$ can be written in frequency domain as:

$$\underline{\mathbf{y}} = \mathbf{S} * \underline{\mathbf{h}} + \underline{\mathbf{n}} \tag{3}$$

which can be useful for channel estimation. Alternatively, we can express the complex conjugate of $\underline{\mathbf{y}}$ as

$$\underline{\mathbf{y}}' = \mathbf{H} * \underline{\mathbf{s}} + \underline{\mathbf{n}}' \tag{4}$$

$\mathbf{H}$ is the channel state matrix in which h_1, h_2, h_3, h_4 and their conjugates form its elements. In Eq. (4), we can recover $\underline{\mathbf{s}}$ from $\underline{y}$ by

09-0107000703©2009 APSIPA. All rights reserved.

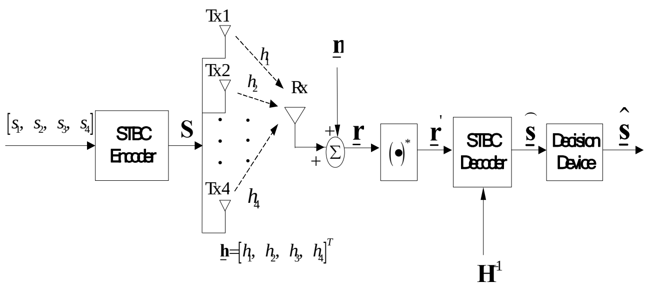

Fig. 2. Basic STBC transceiver model in frequency domain

$$\hat{\underline{\mathbf{s}}} = \mathbf{H}^{-1} * \underline{\mathbf{y}}' \tag{6}$$

For real data symbol, orthogonal STBC matrix can achieve full transmission rate. However, for complex data symbol, such as QPSK, only non-orthogonal STBC data matrix can maintain full transmission rate. There have been two non-orthogonal STBC schemes proposed previously [2,3]. Here, we propose a block-diagonal complex non-orthogonal STBC matrix as

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_2 & s_1 & s_4 & s_3 \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix} \tag{7}$$

We note that $\mathbf{S}^H * \mathbf{S}$ is of block diagonal form:

$$\mathbf{S}^H * \mathbf{S} = \begin{bmatrix} a_6 & b_6 & 0 & 0 \\ b_6 & a_6 & 0 & 0 \\ 0 & 0 & a_6 & b_6 \\ 0 & 0 & b_6 & a_6 \end{bmatrix} \tag{8}$$

where

$$a_6 = \sum_{i=1}^{4} |s_i|^2 \tag{9}$$

$$b_6 = s_1 s_2^* + s_2 s_1^* + s_3 s_4^* + s_4 s_3^* = 2\,\mathrm{Re}[s_1 s_2^* + s_3 s_4^*] \tag{10}$$

$\mathbf{S}^H * \mathbf{S}$ of the other two non-orthogonal STBC matrices are similar except for the locations of the nonzero off-diagonal elements that account for the non-orthogonality between columns of S. Eq. (8) shows that the two non-orthogonal pairs of $\mathbf{S}$ are the 1st and 2nd columns, the 3rd and 4th columns. Since there are only 3 possibilities of non-orthogonal pairs among 4 data columns, our proposed block diagonal form is indeed the only remaining non-orthogonal STBC other than {(1st,3rd)(2nd,4th)} and {1st,4th},(2nd,3rd)} pairings proposed by [2] and [3], respectively.

3. CHANNEL ESTIMATION FOR STBC OFDM

Previously, phase direct (PD) for OFDM in [1] was incorporated with the subspace method [2] in [3] to enhance channel estimation. We will extend this PD technique to BD STBC OFDM and explain why diagonally weighted STBC is necessary for nonsingular channel estimation.

3.1 Diagonally weighted STBC

Consider the mth subcarrier in Eq. (3),

$$\underline{\mathbf{H}}_m = \left[\mathbf{S_m}\right]^{-1} * \underline{\mathbf{y}}_m \tag{11}$$

where $\underline{\mathbf{H}}_m$ is the 4x1 channel frequency response vector at the mth subcarrier, and $\mathbf{S_m}$ and $\underline{\mathbf{y}}_\mathbf{m}$ are transmission matrices corresponding to all possible data and received data vector, respectively.

We note that there exists a problem of $\mathbf{S_m}$ in Eq. (11). For some corresponding $\mathbf{S_m}$ of possible data in BD when BPSK or QPSK is used, $\mathbf{S_m}$ could be singular. In order to prevent $\mathbf{S_m}$ from being singular for all possible data, a diagonally weighted method is therefore proposed as follows:

$$\mathbf{S} = \begin{bmatrix} k*s_1 & s_2 & s_3 & s_4 \\ -s_3^* & k*(-s_4^*) & s_1^* & s_2^* \\ s_2 & s_1 & k*s_4 & s_3 \\ -s_4^* & -s_3^* & s_2^* & k*s_1^* \end{bmatrix} \tag{12}$$

with

$$\mathbf{S}^H * \mathbf{S} = \begin{bmatrix} a_{61} & b_{61} & c_{61} & 0 \\ b_{62} & a_{62} & 0 & -c_{62} \\ c_{61} & 0 & a_{62} & b_{61} \\ 0 & -c_{62} & b_{62} & a_{61} \end{bmatrix} \tag{13}$$

where

$$a_{61} = k^2 |s_1|^2 + \sum_{i=2}^{4} |s_i|^2 \tag{14}$$

$$a_{62} = \sum_{i=1}^{3} |s_i|^2 + k^2 |s_4|^2 \tag{15}$$

$$b_{61} = k(s_1^* s_2 + s_3 s_4^*) + (s_1 s_2^* + s_3^* s_4) \tag{16}$$

$$b_{62} = k(s_1 s_2^* + s_3^* s_4) + (s_1^* s_2 + s_3 s_4^*) \tag{17}$$

$$c_{61} = (k-1)(s_1^* s_3 + s_2^* s_4) \tag{18}$$

$$c_{62} = (k-1)(s_1 s_3^* + s_2 s_4^*) \tag{19}$$

We assume that the diagonal weighting constant k is a real positive number. When $k \neq 1$, the matrix can be shown to be nonsingular for any possible symbol data.

3.2 Phase Direct (PD) --- An improved method for Subspace Channel Estimation

PD is to resolve channel phase ambiguity after getting channel power response. In conventional OFDM system, it is easy to obtain channel power response by simple computation. But in STBC OFDM it is quite different since channel power consists of several different data symbols.

Fig. 3 shows a flowchart of channel power estimation. Suppose we have an initial channel estimate $\mathbf{H}_{est,m}$ obtained from the subspace-based algorithm. Here we aim to find out channel power response on the mth subcarrier by solving the minimization:

$$\min_{\underline{\mathbf{H}}_m = \mathbf{S}^{-1} * \underline{\mathbf{y}}} \| \underline{\mathbf{H}}_{est,m}^P - \underline{\mathbf{H}}_m^P \|^2 \tag{12}$$

in which all possible data symbol vectors $\mathbf{S_m}$, corresponding to channel gain $\underline{\mathbf{H}}_m$ as given in (11), have to be considered in the minimization problem. Here the choice for the *P*-th power depends on the signal constellation. For BPSK *P*=2, and for QPSK, *P*=4. $\underline{\mathbf{H}}_m^P$ denotes the 4x1 channel power vector with each component being taken to its *P*-th power.

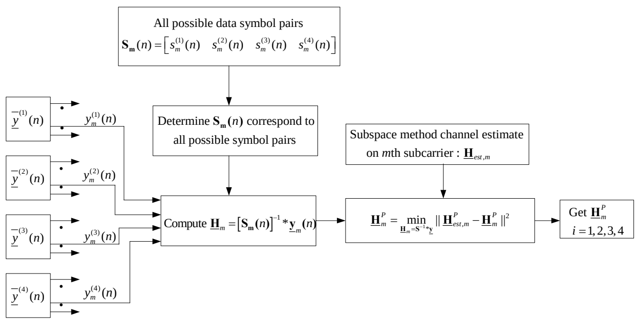

Fig. 3. Channel power response estimation in four-antenna STBC OFDM

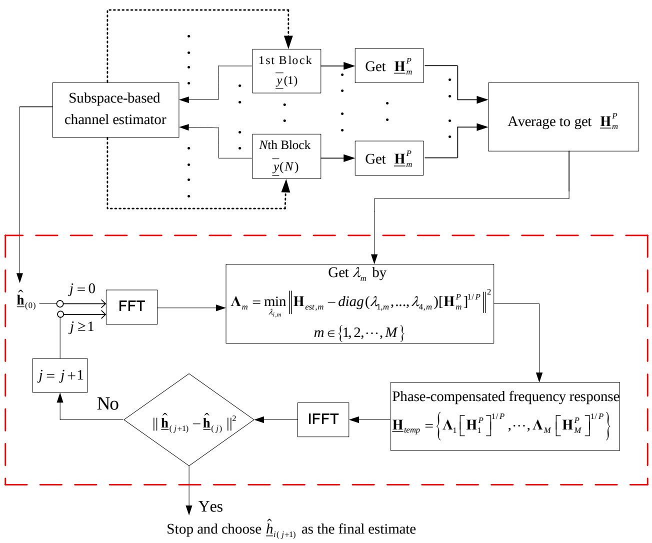

Fig. 4. Phase Direct in four- antenna STBC OFDM

After receiving a total of N data blocks in this algorithm, we can have the time-averaged channel power response**.** The phase ambiguity can be solved by

$$\mathbf{\Lambda}_m = \min_{\lambda_{i,m}} \left\| \mathbf{H}_{est,m} - diag(\lambda_{1,m},\ldots,\lambda_{4,m})[\mathbf{H}_m^P]^{1/P} \right\|^2 \qquad (14)$$

where $\lambda_{i,m} \in \left\{ e^{j2\pi k/P} \mid k = 1,2,\cdots,P \right\}$ on the m th subcarrier for i i th antenna. A 4xM spatial-frequency channel response matrix becomes:

$$\underline{\mathbf{H}}_{temp} = \left\{ \mathbf{\Lambda}_1 \left[\mathbf{H}_1^P \right]^{1/P}, \cdots, \mathbf{\Lambda}_M \left[\mathbf{H}_M^P \right]^{1/P} \right\} \qquad (15)$$

where M is the number of subcarriers. After IFFT, we can perform a denoising process by truncating the time domain channel $\hat{\underline{h}}$. With FFT, a new channel frequency response can replace the initial subspace-based estimate $\mathbf{H}_{est,m}$, which is repeated until $\| \hat{\underline{\mathbf{h}}}_{(j+1)} - \hat{\underline{\mathbf{h}}}_{(j)} \|^2$ converges as shown in Fig. 4.

4. PERFORMANCE ANALYSIS

The theoretical mean square error of channel estimation for the subspace-based method [2] can be written as.

$$E(\| \hat{\underline{\mathbf{h}}} - \underline{\mathbf{h}} \|^2) = E(\| \triangle\underline{\mathbf{h}} \|^2) \approx \frac{\sigma_n^2 \cdot \| \mathbf{Q}^+ \|^2}{\sigma_s^2 N} \qquad (20)$$

where

$$\mathbf{Q} = \begin{bmatrix} \tilde{\mathbf{U}}_q & \tilde{\mathbf{U}}_{nq} \end{bmatrix} \begin{bmatrix} \tilde{\sum}_q & \mathbf{0} \\ \mathbf{0} & \tilde{\sum}_{nq} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{V}}_q^H \\ \tilde{\mathbf{V}}_{nq}^H \end{bmatrix} \qquad (21)$$

and

$$\mathbf{Q}^+ = \tilde{\mathbf{V}}_q \left(\tilde{\sum}_q \right)^{-1} \tilde{\mathbf{U}}_q^H \qquad (22)$$

σ_s^2 / σ_n^2 is SNR. $\mathbf{Q}$ is an important information matrix comprising singular vectors of the received data symbol $\underline{\mathbf{y}}$ [4].

When the diagonal weighting constant k is larger, the diagonal elements of $\tilde{\Sigma}_q$ becomes larger and $(\tilde{\Sigma}_q)^{-1}$ in (22) will become smaller, which will lead to a smaller estimated mean square error in (20).

However, an increased diagonal weight k also increases the transmitted power as well. For same SNR, the noise power σ_n^2 at the receiver needs to be increased, and bit error rate for symbol detection will degrade accordingly.

From Eq. (3), replacing true $\underline{\mathbf{h}}$ with $\hat{\underline{\mathbf{h}}} = \underline{\mathbf{h}} + \triangle\underline{\mathbf{h}}$, we have

$$\underline{\mathbf{y}} = \mathbf{S} * \hat{\underline{\mathbf{h}}} + \underline{\mathbf{n}} = \mathbf{S} * \underline{\mathbf{h}} + \underline{(\mathbf{S} * \triangle\underline{\mathbf{h}} + \underline{\mathbf{n}})} \qquad (23)$$

We can see that correct detection of the transmitted symbol s from the received data $\underline{\mathbf{y}}$ depends on the channel error $\triangle\underline{\mathbf{h}}$ and noise. Suppose they are independent, the equivalent overall perturbation power can be written as

$$P_{\text{total}}(k) = P_{\mathbf{S}*\triangle\underline{\mathbf{h}}}(k) + P_{\underline{\mathbf{n}}}(k) \qquad (24)$$

which is a function of the weighting constant k.

Now we will examine how the increased diagonal weight k decreases the channel estimation error $\triangle\underline{\mathbf{h}}$ but also increases the noise $\underline{\mathbf{n}}$. In case of BPSK with P=2, we note that $E\left[\mathbf{S}^H \cdot \mathbf{S}\right] = \left(k^2+3\right) \cdot I$, then the equivalent channel estimation error power can be shown to be

$$\begin{aligned} P_{\mathbf{S}*\triangle\underline{\mathbf{h}}}(k) &= E\left[| \mathbf{S} \cdot \triangle\underline{\mathbf{h}} |^2 \right] = \left(k^2+3\right) \cdot E(\| \triangle\underline{\mathbf{h}} \|^2) \\ &\approx \left(k^2+3\right) \cdot \frac{\sigma_n^2 \| \mathbf{Q}^+(k) \|^2}{\sigma_s^2 N} \end{aligned} \qquad (25)$$

Notice that $\| \mathbf{Q}^+(k) \|^2$ decreases as k increases**.** Assume that all channels are normalized and uncorrelated, $h_i^* * h_j = \delta_{i,j}$, then the average noise power can be shown to be:

$$P_{\underline{\mathbf{n}}}(k) = \frac{k^2+3}{4} \cdot \frac{\sigma_n^2}{\sigma_s^2} \qquad (26)$$

where σ_s^2 / σ_n^2 is the signal-to-noise ratio.

Finally we have

$$P_{\text{total}}(k) = (k^2+3) \cdot \frac{\sigma_n^2}{\sigma_s^2} \cdot \left[\frac{\| \mathbf{Q}^+(k) \|^2}{N} + \frac{1}{4} \right] \qquad (27)$$

It is now clear that the choice of the diagonal weighting constant k is a tradeoff between channel estimation error in (25) and noise in (26).

5. COMPUTER SIMULATION

Our simulation parameters are: the block size N=100, no. of subcarriers M=32, and four independent 5-ray Rayleigh fading channels. We compare the theoretical and simulated normalized mean square channel errors (NMSCE) for BD STBC BPSK system using the subspace-based channel estimation in Fig. 5. We can see that a large weighting constant *k* improves channel estimation and the simulated curves approach theoretical ones in (20) at high SNR.

Fig. 6 shows the improved NMSCE performance if the subspace-based method is further enhanced by the phase direct technique.

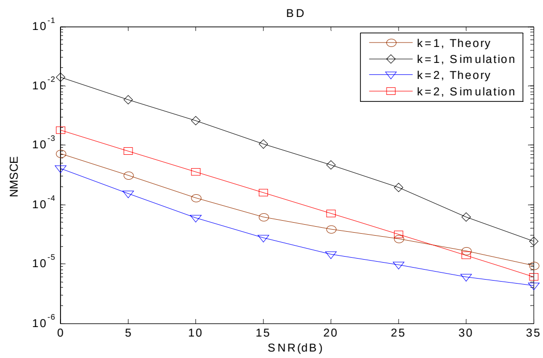

Fig. 5. Theoretical and simulated NMSCE for subspace-based channel estimation

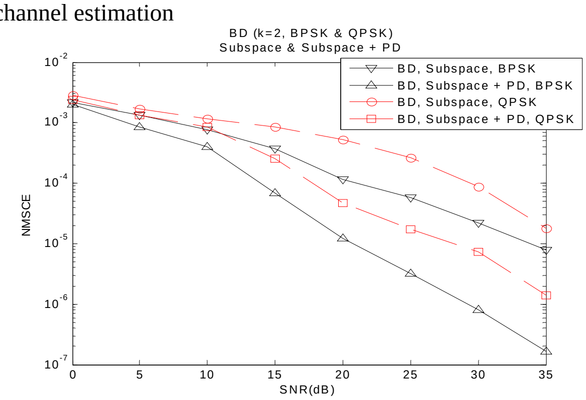

Fig. 6. Improved performances by the phase-direct technique

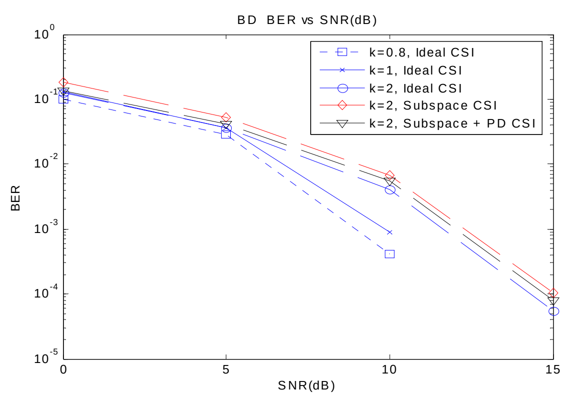

Fig. 7. Bit error rate comparison when k =0.8, 1, 2.

Fig. 7 compares bit error rates when k =0.8, 1, 2. If the channel state information (CSI) is ideal, a larger k degrades the bit error rate. In the case of *k=2,* again, we can see the phase-direct technique approaches the ideal CSI case.

The results of simulated noise power $P_{\underline{\mathbf{n}}}(k)$, equivalent channel estimation error power $P_{\mathbf{S}^*\Delta\underline{\mathbf{h}}}(k)$, both simulated and theoretical total perturbation powers $P_{\text{total}}(k)$ with SNR=10, 15dB are shown in Fig. 8. We can see that as *k* increases, the noise curve increases too, while the equivalent channel estimation error decreases, and the total perturbation power seems to be dominated by the noise power. It appears that the weighting constant can fall into (0.1,0.5) to have the small total perturbation power.

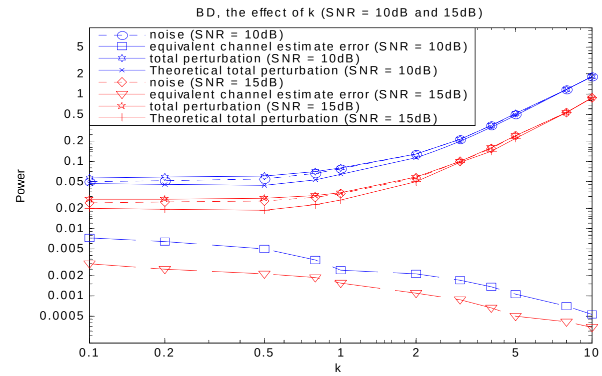

Fig. 8. Theoretical and simulated total perturbation powers

6. CONCLUSIONS

In this paper, we propose a block diagonal non-orthogonal SBTC scheme with diagonal weighting to facilitate the phase-direct channel estimation. We have derived and simulated its mean square channel estimation error and the bit error performances. Large weighting constant improves channel estimation accuracy. However, to compensate the increased transmitted signal power due to an enlarged diagonal weighted signal, the noise power is also increased so that the SNR can remain unchanged for purpose of fair comparison. In such case BER degrades. Simulation indicates a slightly weighting constant with $k \in (0.1,0.5)$ is proper in terms of a better channel estimation and symbol recovery.

7. REFERENCES

[1] T. H. Liew and L. Hanzo, “Space-time codes and concatenated channel codes for wireless communications,” *Proc. IEEE*, Vol. 90, pp. 187-219, February 2002.

[2] O. Tirkkonen, A. Boariu, and A. Hottinen, “Minimal non-orthogonality rate 1 space-time block code for 3+ Tx antennas,” *IEEE 2000 6th Int'l Symp on Spread Spectrum Tech. and Appl.*, Vol. 2, pp. 429-432, September 2000.

[3] R. Ran, J. Hou, and M. H. Lee, “Triangular non-orthogonal space-time block code,” *IEEE 57th Semiannual Vehicular Tech. Conf.*, Vol. 1, pp. 292-295, April 2003.

[4] S. Zhou and G. B. Giannakis, “Subspace-based (semi-) blind channel estimation for block precoded space-time OFDM,” *IEEE Trans. on Signal Processing*, Vol. 50, pp. 1215-1228, May 2002.

[5] S. Zhou and G. B. Giannakis, “Finite-alphabet based channel estimation for OFDM and related multicarrier systems,” *IEEE Trans. on Communications*, Vol. 49, pp. 1402-1415, August 2001.

[6] S. F. Hsieh and T. Y. Wu, “Phase-direct channel estimation for space-time OFDM,” *ICASSP*, Vol. IV, pp. 369-372, May 2006.