

A Game-Theoretical Model of Cognitive Relay

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Abstract—Cognitive Radio provides enhanced wireless communication performance through context-aware sensing and adaptation. Cooperative relay enhances network connection quality and improved system throughput. Applying cooperative relay communication technique on Cognitive Radio networks enables Cognitive Radio to opportunistically exploit cooperation between network nodes. This integrated design that benefits from both cognitive network operation and cooperative communications. One of the key challenges in the Cognitive Radio based cooperative relay network is to seek optimized network node cooperation in a distributed environment. Recently, game theory has been applied to solve communications and networking problems. In this paper, we adopt a game theoretic approach to facilitate the cooperative relay in Cognitive Radio networks.

I. INTRODUCTION

Cognitive Radio, which was first introduced by J. Mitola [5], is a promising technology for building next generation wireless technology. It offers the flexibility for users to organize their transmission parameters (such as frequency and power) or behavior (idle, transmit, or relay) to fulfill their needs in wireless communication. The fundamental idea of Cognitive Radio is sense-then-act process. Users in a Cognitive Radio network sense the network environment to derive the knowledge of wireless network. According to the knowledge, they decide their behaviors and optimize their parameters. A learning algorithm is implemented for users to "learn" from the repeated process to fulfill their needs, like maximization on transmission throughput, minimized interference environment, or maintain QoS requirement.

Underlay, overlay and interweave approach are commonly mentioned in Cognitive Radio system model [7]. Underlay approach protects primary user with the help of ultra wideband (UWB). In overlay approach, users allocate partial power to transmit primary users' messages to maintain or even enhance the SNR of primary users. For interweave approach, which we focused on in this paper, primary users and secondary users should never transmit at the same time. Secondary users are only allowed to transmit while primary users are idle. In this approach, secondary transmitters and receivers should carefully detect the activities of primary users to avoid interference. Dynamic spectrum access is a popular issue in this approach [1]. A statistic model on channel access has been proposed in [2] to efficiently predict the spectrum access scheme of the primary users. If there are multiple channels available, an Opportunistic Channel Selection problem is arisen. The capacity of this kind of Cognitive Radio network has been discussed in [3]. In addition, a channel selection design with partial observation has been proposed in [8].

A. Cognitive Relay

Dealing with the coexistence of primary users and secondary users, O. Simeone *et al.* design a cooperation scheme among users [6]. Cognitive Relay approach is introduced in this paper to enhance the throughput of primary users while increasing the transmission opportunities of secondary users. They discussed a simplified scenario: only one primary pair (one transmitter and one receiver) and one secondary pair coexist in the system. It is assumed that the channel gain between primary pair is smaller than secondary transmitter with each others. A secondary transmitter may help relaying primary user's packet if the transmission is detected failed. Because of higher channel gain, the transmission is more likely to successes with the relay of secondary user. With Cognitive Relay, primary users can prevent unnecessary retransmission, and secondary users can improve their transmission opportunities while keeping transparent to primary users. The simulation results showed a significant improvement on the throughput of all users with Cognitive Relay mechanism.

Extended from Simeone's work, the goal of our research is creating a generalized mechanism for Cognitive Relay in interweave approach. A Cognitive Relay model with multiple secondary users is introduced first with additional concerns on battery power and data transmission. To discuss the behaviors of users in the generalized form, a Nash Game model is applied to analyze the strategies these secondary users will use. The Nash Equilibrium in our Nash Game model has been derived, and the performance enhancement with Cognitive Relay is studied with simulations.

II. SYSTEM MODEL

We consider a Cognitive Radio network with one primary sender, one primary receiver, and multiple secondary users with battery concerns. We define the primary sender as P_s , the primary receiver as P_r , and N secondary users $S_i \in S$ are located around P_s and P_r . We assume they all have the ability to sense one's transmission in this network, which means no hidden terminal problem. An illustration of our system model is shown in Fig. 1.

For primary users, the packet error rate (PER) from P_s to P_r is defined as e_0 . e_0 is affected by the coding scheme, channel gains, and interference. For secondary users, the PER from S_i to P_r is defined as e_i for $1 \leq i \leq N$. Because of the different locations and channel qualities of secondary users, their PER varies largely. Those which are closer to P_r or with less interference may have smaller PER than e_0 , and vice versa. The transmission time in our system model is slotted

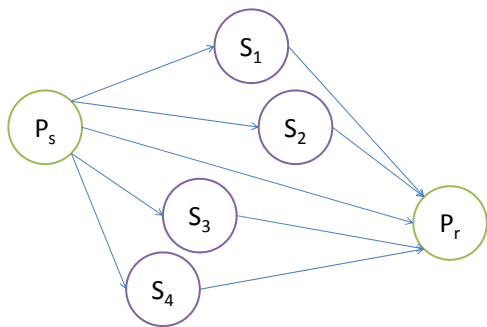


Fig. 1. System Model

with slot time T_s . Every data transmission from both primary and secondary users have the same and fixed transmission time within one slot, including the overhead such as RTS/CTS and ACK. We assume P_s 's traffic scenario is constant-bit-rate (CBR) with time interval T_{inv} . As a Cognitive Radio network, P_s has the license to access the channel without any interference from secondary users. Secondary users have the right to access the channel when there is no active primary user. In our system, the only chance secondary users can access the channel is the successive slots after P_r receive the data from P_s successfully. In this case, secondary users can use the rest of time slots for transmission until P_s has another CBR packet for transmission and start accessing the channel.

A. Cognitive Relay Process

When the packet transmission from P_s fails with probability e_0 , a Cognitive Relay process is triggered to recover the failure. In the process, P_s always choose to perform its own retransmission process unless P_s senses that another user has help relaying the packet. For user S_i which have overheard it, whether it performs Cognitive Relay or not depends on its own sense-observation-decision process in cognitive fashion. We define the set $R' \subset S$ as the set of secondary users who decide to perform Cognitive Relay and $n(R') = n - 1$. In a relay process, n users (including P_s) challenge for the relay opportunity. We assume the probability for each user of getting the relay opportunity is equal to $1/n$. This can be realized with a RTS-CTS fashion random access with same CW value. If the retransmission or Cognitive Relay fails, All users in $R = R' \cap \{P_s\}$ challenge repeatedly until there is a secondary user S_i or P_s successfully transmits the packet to P_r , and the transmission failure is recovered. We define the time of relay process as T_r , and the remain time $T_s = T_{inv} - T_r - 1$ is the transmission time for all secondary users in S .

B. Nash Game Formulation

Game theory is naturally suitable for analyzing the Cognitive Radio [4]. A proper game theory framework can offer a distributed user view on wireless network, and users can be characterized in cooperative or selfish manners, and their actions (behaviors) are based on their knowledge to the game.

These characteristics perfectly fit the requirements of modeling Cognitive Radio architecture.

In Cognitive Relay process, P_s must join R in order to ensure at least one user retransmits the packet. In contrast, a secondary user can choose whether to relay the overheard packet or not. We consider the users in the network are selfish, which means their choices depend only on the benefits they can earn from. Considerations on the social welfare and optimization issues are ignored. In addition, a centralized optimization mechanism may not be suitable for distributed wireless users, which is the case in our system model.

Following from the discussion above, we need to formulate the behaviors of distributed secondary users and discuss a practical distributed mechanism to organize the secondary users in our model. A Nash Game Model is suitable for all the requirements. We define the players as all secondary users $S_i \in S$, each with only two actions can be choose from: relay or not relay, in other words, join R or disjoint R . The action space of S_i can be written as $\{S_i \in R, S_i \notin R\}$. A strategy of the player is defined as a function of network information, and its output is the action player chooses. The action it chooses depends on the expected utility it can gain from the decision. If it is more beneficial for the user to perform Cognitive Relay, it will choose to join R , otherwise it will refused to relay. We use the concept of "satisfaction - sacrifice" to define player S_i 's utility. For every successful transmission of its own data, the utility pluses d_i , which represent the satisfaction rate for a successful data transmission. For every transmission (including relay), the utility minus b_i , which represent the sacrifice rate of using its own battery power for transmission. So the utility function of S_i can be described as follows:

$$u_i = d_i([\text{number of } S_i\text{'s own data transmission}]) - b_i([\text{number of } S_i\text{'s total transmission}]) \quad (1)$$

C. Cognitive Relay Scheme

We proposed a relay process scheme in our Cognitive Radio system. After a successful transmission of packet to P_r , the remain transmission time T_s is shared by all secondary users in a random access fashion. They choose their strategy to maximize their utility. This is applied in the network with Cognitive Relay scheme discussed in [6].

III. STRATEGY ANALYSIS

We assume all secondary users are selfish and the only target in their mind is to maximize their own expected utilities. The choice of relay or not depends on the expected utilities they can derive from the choice. In this section, we first discuss the influence of their choice on the network performance, like average packet error and expected total relay time. After analyzing the influence, we discuss the benefit of an arbitrary secondary user from its choice of strategy.

The strategy which user chooses should optimize their expected utility. We defined \bar{e} as the average relay PER of the primary packets. The expected relay time of a primary

packet with \bar{e} is

$$E[T_b] = \sum_{k=1}^{\infty} k\bar{e}^{(k-1)}(1-\bar{e}) = \frac{1}{1-\bar{e}} \quad (2)$$

Which is only related to the \bar{e} . When \bar{e} is high, the relay time increases significantly. For every users in R , we know that their expected relay time (the transmission time they expect to spend in the relay process) T_{b_i} is

$$E[T_{b_i}] = E[T_b]/n(S_r) = E[T_b]/n = \frac{1}{n(1-\bar{e})} \quad (3)$$

With the discussion of T_b and T_{b_i} , we know that for every users in R , they have equal probabilities to relay and the same expected relay time. When a user with larger PER join R , the average PER increases and all users are suffered by longer expected relay time. On the other hand, if a user with small PER join R , the average PER decrease and all users are benefit from the reduction on the relay time. This is the key factor for users to decide their best strategy.

We can write down the utility of S_i with relay action:

$$u_{i_r} = -b_i\left(\frac{1}{n(1-\bar{e})}\right) + (d_i - b_i)\left(\frac{T - \frac{1}{1-\bar{e}}}{N}\right) \quad (4)$$

And the utility of S_i with no relay action is:

$$u_{i_n} = (d_i - b_i)\left(\frac{T - \frac{n-1}{n-n\bar{e}-(1-e_i)}}{N}\right) \quad (5)$$

For a selfish user S_i , the best strategy for S_i is to relay when $u_{i_r} > u_{i_n}$. Otherwise, S_i choose not to relay. To make things more clear, we rearrange the equation as

$$\begin{aligned} u_{i_r} - u_{i_n} &= -b_i\left(\frac{1}{n(1-\bar{e})}\right) + (d_i - b_i)\left(\frac{T - \frac{1}{1-\bar{e}}}{N}\right) \\ &\quad - (d_i - b_i)\left(\frac{T - \frac{n-1}{n-n\bar{e}-(1-e_i)}}{N}\right) \geq 0 \\ \Rightarrow d_i/b_i &\geq 1 + \frac{N(n - n\bar{e} - (1 - e_i))}{n(\bar{e} - e_i)} \quad (\text{for } \bar{e} > e_i) \end{aligned} \quad (6)$$

From (6) we first observed that $\frac{N(n - n\bar{e} - (1 - e_i))}{n(\bar{e} - e_i)}$ represent the ratio of average successful probability of $R - \{S_i\}$ to the gap between \bar{e} and e_i . We have following conclusions from this equation:

- 1) If the average successful probability exclude S_i is large, the threshold of choosing to relay will be larger. This is reasonable because the join of S_i is less beneficial for relay process. The reduced relay time may not large enough to recover its own cost on battery power.
- 2) If the gap between \bar{e} and e_i is positive and large, S_i is more likely to relay, because it is more significantly beneficial to overall relay process on the reduction of the average PER.
- 3) If $\bar{e} < e_i$, Not to relay is the dominate strategy of S_i . This is observed from the equation rearrangement in (6). When $\bar{e} > e_i$, the threshold will become negative and the inequality is reversed. As a result, any user S_i with positive data/battery ratio will not satisfy (6). This is also

a reasonable result due to the join of S_i with $e_i > \bar{e}$ is no beneficial for all users because the join of S_i increase average PER.

Another observation we have on (6) is that the number of total users (N) is a critical issue because the threshold increases linearly with N . This means when more secondary users are in the same area, their incentive to choose the relay action is reduced. This is because the benefits from reduced relay time is shared not only by the users in R but also by all the secondary users. For users with small PER, they are more unlikely to join R even if their join can increase relay success probability.

IV. NASH EQUILIBRIUM

In a Nash Game, the existence of Nash Equilibrium is an important issue because it is related to the stability of the game and system. In our game model, we can prove by theory that when all secondary users's d_i/b_i ratio are equal, there exists a Nash Equilibrium. The proof and the method for finding this equilibrium is a solution to a simple optimization problem with constraints.

Proposition 1: *If all secondary users have the same $d_i/b_i = d$ ratio, a Nash Equilibrium exists and can be derived with a $o(n \log n)$ complexity algorithm.*

Proof: In the beginning of the process, we only add one user P_s into R . We sort the secondary users with their PER. In the first round, we check the best strategy of the secondary user S_j with minimum PER. If the best strategy of S_j is join R , we add S_j into R . Then we repeat this process with remaining secondary users in $N - R$ until the secondary user with minimum error rate S'_j rejects to join R .

Now we examine if all the secondary users in R will not change their strategy. We define S_i as the secondary user with maximum PER e_i in R . S_i is the last secondary user who joins R . According to the process, e_i must satisfy the following equation.

$$d_i/b_i = d/b \geq \frac{N(n - n\bar{e} - (1 - e_i))}{n(\bar{e} - e_i)} \quad (7)$$

For those secondary users S_j in R , $i \neq j$, we know $e_j \leq e_i$ because of definition of e_i . It is easy to check that for all $e_j \leq e_i$, the inequality from above still holds because they all have the same data/battery ratio. We conclude all secondary users in R will choose to relay and stay in the set R . We can have similar discussion on the users in $S - R$ and reaches the conclusion that all users in $S - R$ will not choose to relay and stay in $S - R$.

We proved that the best strategy of users in R is choosing to relay, while the users in $S - R$ is not to relay. This means all users will not change their strategy in the end of this process, and a Nash Equilibrium is derived. The sort process requires $o(n \log n)$ while the check process requires $o(n)$, so this algorithm requires $o(n \log n)$.

For the case of variety of d_i/b_i ratios, our simulations shows that there always exists at least one Nash Equilibrium. From the simulation results, we observed that those who join R mostly have low PER and high d_i/b_i ratio.

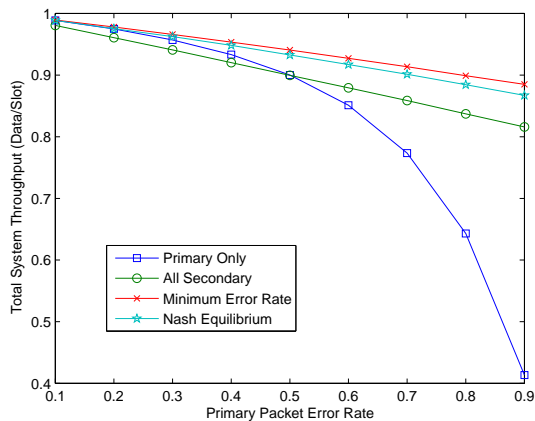


Fig. 2. Simulation Results

V. SIMULATION RESULTS

In order to show the performance enhancement derived from Nash Equilibrium strategy, we setup a Cognitive Radio network scenario and compare the performance of different transmission strategies. We compare Nash Equilibrium strategy with three different strategies: Primary Only, All Secondary, and Minimum Error Rate. In Primary Only strategy, only primary sender is responsible for primary data retransmission, and all secondary users will not help to relay packets. This strategy represents a traditional scheme that primary users and secondary users do not cooperate at all. In All Secondary strategy, all secondary users are forced to help relay the packets from the primary sender. This is a simple scheme with the assumption that Cognitive Relay process is always triggered by all secondary users. In Minimum Error Rate strategy, only the selected lowest error rate secondary users are forced to help relay the packets in order to minimize the packet error rate of primary packets. We apply this strategy as a benchmark of centralized-control mechanism in this network. Finally, we implement our distributed algorithm into secondary users and define this as the Nash Equilibrium strategy.

In the simulation, two primary users are transmitting data with error probability e_0 , and 30 secondary users with error probabilities uniformly distributed between $[0, 1]$ and data/battery ratio uniformly distributed between $[0, 150]$. The traffic scheme of primary user is one packet per 10 slots. Every packet transmission or relay takes exact 1 slot. We simulate 10000 rounds with 4 different strategies: Primary Only, All Secondary, Minimum Error Rate, and Nash Equilibrium from our algorithm. We adjust the PER of primary sender e_0 from 0.1 to 0.9 and measure the system throughput. The results are shown in the Figure 2.

In Figure 2, we observed that when the PER of primary sender increases, the performance of Primary Only and All Secondary strategies degrades significantly. The throughput under Primary Only strategy degrades exponentially because no secondary users help relay the packet. The primary sender

needs more slots for retransmission, and secondary users have less opportunities to transmit their own data. For All Secondary strategy, although the relay of secondary users can recover the high primary PER, the system throughput may degrade because some secondary users with high PER are forced to join the relay process. We observed that for the system that primary PER less than 0.5, All Secondary strategy results worst performance. On the other hand, Minimum Error Rate strategy leads to highest system throughput because it is a centralized-optimal mechanism and the PER of primary packets is minimized. The system throughput of Nash Equilibrium strategy we proposed approximates to that of Minimum Error Rate, and outperforms other strategies. Last but not the least, Nash Equilibrium strategy is stable because no one will unilaterally change his decision even if they are selfish users, which is not promised in Minimum Error Rate strategy. For this reason, the Nash Equilibrium found by our algorithm is indeed the best strategy which conforms to the system requirements.

VI. CONCLUSION

We introduced a generalized system model for Cognitive Relay and formulate a Nash Game model based on this system model. We proposed a Cognitive Relay scheme for this Nash Game model. We formulated a centralized feasibility problem for Nash Equilibrium based on the system model and transformed the problem into multiple distributed optimization problem on each secondary user. The analysis of Nash Equilibrium showed that the behaviors of secondary users are greatly influenced by other users and environments. We simulated and compared different strategies of Cognitive Radio network, and showed that the Nash Equilibrium of our proposed Cognitive Relay scheme enhances the performance of Cognitive Radio network.

VII. ACKNOWLEDGEMENT

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