

# Channel-Aware Reservation-Based MAC Protocol for Cognitive Radio Networks

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**Abstract**—A channel-aware reservation-based MAC protocol for cognitive radio networks (CRNs) is proposed in this work. The proposed scheme allows secondary users (SUs) to send out reservation requests at the beginning of each time slot in a distributed fashion according to the sensing outcomes and their local channel state information (CSI). A channel-aware splitting algorithm is used to resolve the collision between the reservation requests of different SUs in a way such that the transmission time slot is always assigned to the SU with the best link quality. By exploiting multiuser diversity with the proposed scheme, we show that significant improvements in throughput can be observed. Furthermore, by modeling the presence and absence of the primary users (PUs) using a continuous time Markov chain, the spectrum sensing error and the reliability of the sensing outcomes across time are considered when designing the MAC policy. The effect of the data transmission and the spectrum sensing periods on the system throughput is studied via numerical simulations.

## I. INTRODUCTION

Radio spectrum is a scarce resource for wireless communications and mobile applications. Most of the spectrum has been allocated to different usages or is licensed to specific users. However, the use of these licensed frequency bands have been observed to be extremely inefficient, leaving many parts of the spectrum vacant over long periods of times [1]. To enhance the spectrum utilization, *opportunistic spectrum access* [2] policies have been proposed for cognitive radio networks where secondary networks of unlicensed users operates opportunistically in the vacant spaces of the spectrum that originally belongs to a group of primary or licensed users. In order to protect the primary users (that is, the licensed users), secondary users are required to sense the spectrum and then determine the spectrum access probability so that the interference or collision experienced by the primary user is maintained below a certain level.

Recently, several OSA policies have been proposed by taking into consideration the reliability of spectrum sensing, e.g. [3][4][5]. By modeling the spectrum occupancy state of the primary network as a two-state Markov model, the optimal spectrum sensing and channel access strategies have been derived in [3] for multichannel systems using Markov decision process. A constrained partially observable Markov decision process (POMDP) is proposed in [4] for the SU to derive the

optimal spectrum sensing and access policies with spectrum sensing error. For the secondary networks with multiple SUs, a decentralized channel access policy is proposed in [5] to avoid the collision between the secondary users. However, in cognitive radio networks, the SUs can only utilize the channel opportunistically when the PU is not present. The time available for the SUs to transmit may be extremely limited and, thus, an efficient use of the channel is critical. In this work, we study how the SUs can utilize local channel state information (CSI) and spectrum sensing outcome to derive an efficient spectrum access policy. The use of CSI allows for exploitation of multiuser diversity among SUs.

Multiuser diversity [6][7] has been exploited in the past, for both uplink and downlink systems, by scheduling users with the best channel to transmit in each time slot, which can be applied in both centralized and decentralized systems. Although centralized channel-aware scheduling policies achieve high system throughput, the overhead and delay involved (particularly in the collection of CSI from the users) may prohibit the use of these strategies in highly dynamic environments or networks with large number of users. However, decentralized implementation of the concept, such as the so called channel-aware slotted ALOHA random access [8][9][10], is not suitable for the networks with heavy load. As a compromise between these two extremes, a reservation-based channel-aware slotted ALOHA transmission control can be adopted, similar to that in [11] where only users with sufficiently good channels are allowed to reserve the channel.

The main contribution of this work is to propose a channel-aware reservation-based MAC protocol for secondary networks that consist of multiple SUs. Consider the scenario that the secondary network performs spectrum sensing periodically and the secondary users transmit according to the sensing outcomes and their CSI in a decentralized reservation-based fashion. More specifically, each SU with data to transmit will opportunistically reserve a transmission time slot according to its local CSI and spectrum sensing outcome. Based on the channel-aware splitting algorithm [12] with which the collision among reservation packets sent by different users is resolved and the user with the best channel quality is always selected to transmit, a modified splitting algorithm is developed in this work which considers not only the CSI at secondary users but also the spectrum sensing outcomes such that the probability of colliding with PU signals is restricted. Also, SU's decision on whether or not to transmit depends on the reliability of

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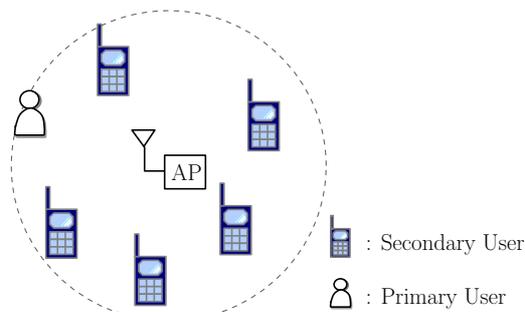


Fig. 1. Network model.

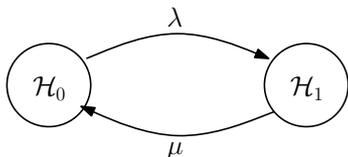


Fig. 2. Transmission model for PU.

the sensing outcome which decreases with time since that the state of the primary network changes with time. Hence, as the sensing outcome becomes less reliable over time, the transmission strategy of SUs will become conservative due to the uncertainty of the spectrum state. The impact of the time-varying reliability of sensing outcomes on the network throughput will be studied via numerical simulations.

The rest of the paper is organized as follow. The system model is described in Section II. The channel-aware reservation-based MAC protocol with the proposed modified splitting algorithm is introduced in Section III. The analysis on the network throughput is given in Section IV by considering a collision constraint to the primary network. Finally, some simulation results are shown in Section V and the conclusion goes in section VI.

## II. SYSTEM MODEL

Suppose that there are  $L$  secondary users, *i.e.* SU 1 to SU  $L$ , communicating with the access point (AP) as shown in Fig. 1, and that the secondary network lies within the coverage of the primary network. The presence or absence of the PU signals can be modeled as a two-state continuous-time Markov process [3] as shown in Fig. 2. The spectrum occupancy state can be *idle* (state  $\mathcal{H}_0$ ) or *busy* (state  $\mathcal{H}_1$ ) with the transition rate matrix

$$Q \triangleq \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix},$$

where  $1/\lambda$  (or  $1/\mu$ ) is the mean holding time that the PU is idle (or busy). In other word, each PU transmission will last for  $1/\mu$  seconds on the average and then, keep silent for about  $1/\lambda$  seconds. Hence, the probability that the PU transmission is idle (or busy) is denoted by  $v_0$  (or  $v_1$ ), where

$$v_0 = \frac{\mu}{\mu + \lambda}, \quad v_1 = \frac{\lambda}{\mu + \lambda}.$$

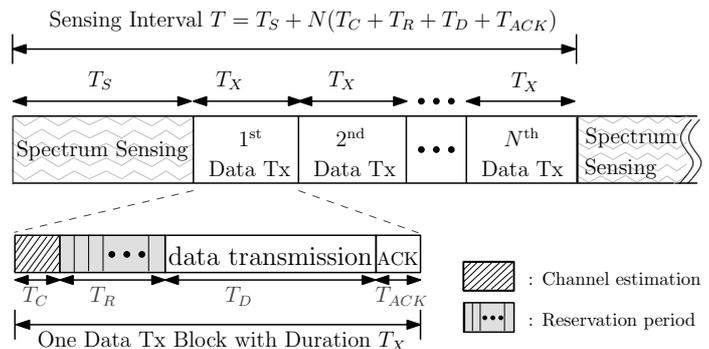


Fig. 3. Time slot model.

Suppose that all the  $L$  SUs have data to transmit to the AP. The AP will perform the spectrum sensing either by itself or with the help of other SUs. Then, the spectrum occupancy state ( $\mathcal{H}_0$  or  $\mathcal{H}_1$ ) will be determined at the AP and the performance of the spectrum sensing can be characterized with two sensing error probabilities, that is, the false-alarm probability  $P_{FA}$  and the miss detection probability  $P_M$ , where

$$P_{FA} = \Pr\{\mathcal{H}_1 \text{ detected} | \mathcal{H}_0 \text{ true}\}$$

and

$$P_M = \Pr\{\mathcal{H}_0 \text{ detected} | \mathcal{H}_1 \text{ true}\}.$$

Since that the spectrum occupancy state changes with time, the error probabilities here are defined with respect to the actual spectrum occupancy state at the end of the spectrum sensing period.

After the spectrum sensing,  $N$  consecutive data transmission blocks are initiated for the SUs to transmit as shown in Fig. 3. Each transmission block with duration  $T_X$  is composed with four phases, *i.e.* the channel estimation, the reservation, the data transmission, and the ACK reception phase, and each of the phases occupies with duration  $T_C$ ,  $T_R$ ,  $T_D$ , and  $T_{ACK}$  respectively. Let  $\gamma_i[n]$  be the channel gain of SU  $i$  in the  $n$ -th transmission block and be exponentially distributed with density

$$f_i(\gamma_i) = \frac{1}{\sigma^2} \exp\left(-\frac{\gamma_i}{\sigma^2}\right),$$

which means that the channel coefficient between each SU and the AP is a complex Gaussian random variable with zero mean and variance  $\sigma^2$ . Moreover, the channel gain of each SU is assumed to be identical and independently distributed (*i.i.d.*) across time blocks and is also *i.i.d.* among SUs. Thus, we shall omit the user index  $i$  of the random variable  $\gamma_i$  in the following discussion. In order to enhance the network throughput by exploiting the multiuser diversity, the AP is supposed to serve the SU with the highest channel gain in each transmission block. However, the AP does not know the exact channel gains between itself and all SUs. In fact, to collect all the channel state information from all SUs will take large amount of effort especially when the number of users goes to infinity.

In this work, we are considering a channel-aware reservation-based MAC protocol with collision resolution,

where the AP is not required to collect all the SUs' CSI but just allocates some minislots for SUs to reserve their transmission. The reservation scheme will be particularly described in Section III. Intuitively, all SUs estimate their channel gains between themselves to the AP at the beginning of each transmission block, where the channel estimation can be done with the help of the pilot transmitted from the AP. Thereupon, the SUs with larger channel gain will emit the transmission requests in order to reserve the time for transmission. The user who has successfully reserved the transmission without colliding with other SUs' request will transmit in the following data transmission period. The AP will then feedback an positive ACK packet if it correctly decodes a packet from one of the SUs. Besides, if there is more than one SU emitting the transmission request, the requests will collide and the AP will yield a collision such that the collision resolution algorithm can then be applied among the collided users. Furthermore, if no SU requests for transmission, the AP will not send any ACK messages.

However, with spectrum sensing error, the transmission probability of SUs in each transmission block should be carefully determined based on the spectrum sensing result as well as the time elapsed after the spectrum sensing. Let define that

$$b_i[n] = \Pr \{ \text{PUs are idle during block } n \text{ given that } \mathcal{H}_i \text{ true} \},$$

for  $i = 0, 1$ , which depends on the spectrum occupancy state obtained in the spectrum sensing period. By considering the transmission of the primary network as a continuous-time Markov process, the spectrum idle probabilities are given by [3]

$$b_0[n] = \exp(-\lambda T_X) \left\{ 1 - \frac{\lambda}{\lambda + \mu} [1 - \exp(-(\lambda + \mu)t[n])] \right\},$$

and

$$b_1[n] = \exp(-\lambda T_X) \frac{\mu}{\lambda + \mu} [1 - \exp(-(\lambda + \mu)t[n])],$$

where  $t[n] = (n-1)T_X$  is the time elapsed after the spectrum sensing period until the beginning of the  $n$ -th transmission block. Specifically,  $\left\{ 1 - \frac{\lambda}{\lambda + \mu} [1 - \exp(-(\lambda + \mu)t[n])] \right\}$  (or  $\frac{\mu}{\lambda + \mu} [1 - \exp(-(\lambda + \mu)t[n])]$ ) denotes the probability that the primary network is idle at the beginning of  $n$ -th transmission block given that the spectrum is idle (or busy) in the spectrum sensing period. In addition,  $\exp(-\lambda T_X)$  denotes the probability that the spectrum remains vacant during a time period  $T_X$  given that the spectrum is idle in the beginning of the period.

Since that the transmission state of PU signals changes with time, we say that a collision occurs in one transmission block such that the transmission of secondary networks fails once the PU signals become busy during the block. In order to prevent from colliding with PU signals, the SUs shall limited the transmission probability of secondary networks in each transmission block. Let denote  $p_j[n]$  as the probability that the secondary network transmits in the  $n$ -th transmission block given that the spectrum state  $\mathcal{H}_j$  is detected. Hence, the

probability that the transmission of SUs collides with the PU signals given that the primary network is transmitting is given by

$$\begin{aligned} & \Pr \{ \text{SU transmits} | n\text{-th block is not idle} \} \\ &= \frac{p_0[n]c_0[n] + p_1[n]c_1[n]}{v_0(1 - b_0[n]) + v_1(1 - b_1[n])}, \end{aligned} \quad (1)$$

for  $n = 1, 2, \dots, N$ , where

$$c_0[n] = v_0(1 - P_{FA})(1 - b_0[n]) + v_1P_M(1 - b_1[n])$$

is the probability that  $\mathcal{H}_0$  is detected in the spectrum sensing period and the transmission of SUs is collided with the PU signals in the  $n$ -th transmission block. Similarly,

$$c_1[n] = v_0P_{FA}(1 - b_0[n]) + v_1(1 - P_M)(1 - b_1[n])$$

is the probability that  $\mathcal{H}_1$  is detected and the transmission of SUs and PU signals are collided in the  $n$ -th transmission block. In order to exploit the multiuser diversity in the secondary networks, we adopt a transmission strategy which is analogous to the case in slotted ALOHA network with channel awareness [8], [9] by setting a threshold  $\Gamma_j[n]$  on the channel gain in the  $n$ -th transmission block if  $\mathcal{H}_j$  is detected, and letting only the SUs whose channel gains greater than  $\Gamma_j[n]$  have the opportunity to reserve the transmission via the proposed modified splitting algorithm in section III. The transmission probability of the secondary network is derived as a function of the threshold, *i.e.*  $\Gamma_0[n]$  and  $\Gamma_1[n]$ . The optimal thresholds that maximize the network throughput are discussed in section IV.

### III. RESERVATION SCHEME FOR SECONDARY USERS

#### A. An Overview of Channel-Aware Splitting Algorithm

In each transmission block, the AP allows SUs to emit the reservation requests in the reservation period, which is composed of  $K$  successive minislots with duration  $T_{min}$  each as shown in Fig. 4. Assume that the number of minislots  $K$  is less than the number of total SUs  $L$ . Otherwise, the AP would have enough time to query each SU about the CSI in different minislot so that SUs need not to perform the reservation. Suppose that the total number of SUs, *i.e.*  $L$ , is known at each SU, a splitting algorithm is used for users to resolve the collided reservation packets. Specifically, to find out the user with the highest channel gain, a query range on the channel gain will be dynamically determined minislot by minislot such that only the users whose channel gains fall within the query range can emit the transmission requests in the corresponding minislot. If there is more than one user requesting for transmission, the AP will yield a collision and then narrow down the query range in order to find the one with the highest channel gain. On the other hand, if there is no one whose channel gain falls within the query range, the AP will move the query range to the lower part. The proposed reservation MAC protocol in this work will adopt a modified splitting algorithm where the query range is set based on not only the CSI of SUs but also the spectrum sensing outcomes for the cognitive radio networks as follow.

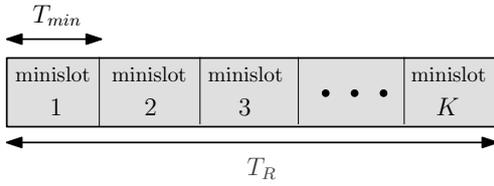


Fig. 4. Reservation period in each transmission block.

### B. Modified Splitting Algorithm for CRN

Consider the cognitive radio networks as described in section II. Assume that the channel-aware transmission strategy and the reservation scheme depend on the spectrum sensing outcomes. That is, if the spectrum is sensed to be idle, then the AP will tend to allow only the users whose channel gain greater or equal to  $\Gamma_0[n]$  to reserve for the transmission in the  $n$ -th data transmission block. On the other hand, only the users whose channel gains greater or equal to  $\Gamma_1[n]$  can reserve for the  $n$ -th transmission block whenever the spectrum is sensed to be busy. The thresholds  $\{\Gamma_0[n]\}_{n=1}^N$  and  $\{\Gamma_1[n]\}_{n=1}^N$  should be carefully chosen so that the collision probability to the primary user is bounded below. Hence, given that  $\mathcal{H}_j$  is detected, the lower bound of the query range should never lower than  $\Gamma_j[n]$  in the  $n$ -th transmission block.

Specifically, in the beginning of each transmission block, the query range is initialized as  $[\max\{\Gamma_j[n], H_{j,1}[n]\}, \infty)$  in the first minislot given that  $\mathcal{H}_j$  is detected, where  $F(H_{j,1}[n]) = 1 - \frac{1}{L}$ . The function

$$F(\gamma) = 1 - \exp\left(-\frac{\gamma}{\sigma^2}\right)$$

is the cumulative distribution function of the channel gain. Note that,  $H_{j,1}[n]$  is chosen to maximize the probability that only the user with the highest channel gain is queried in the first minislot. The maximization between  $\Gamma_j[n]$  and  $H_{j,1}[n]$  ensures that the query range is all above  $\Gamma_j[n]$ . Given that the previous  $i - 1$  minislots are idle, *i.e.* all the SUs' channel gains are below the value of  $H_{j,i-1}[n]$ , the lower bound of the query region in the  $i$ -th minislot, *i.e.*  $H_{j,i}[n]$ , will be updated in order to maximize the probability that the highest channel gain can be resolved in the next minislot, that is,

$$H_{j,i}[n] = \arg \max_H \frac{\binom{L}{1} (F(H_{j,i-1}[n]) - F(H)) [F(H)]^{L-1}}{[F(H_{j,i-1}[n])]^L}.$$

Hence, after  $i - 1$  successive idle minislots, the query range in the  $i$ -th minislot is set to be

$$[\max\{\Gamma_j[n], H_{j,i}[n]\}, \max\{\Gamma_j[n], H_{j,i-1}[n]\}],$$

where

$$F(H_{j,i}[n]) = \left(1 - \frac{1}{L}\right)^i. \quad (2)$$

While the first non-idle minislot is found, let say the  $i$ -th minislot, it is known that there is at least one SU whose channel gain falls within  $[\max\{\Gamma_j[n], H_{j,i}[n]\}, \max\{\Gamma_j[n], H_{j,i-1}[n]\})$ . If more

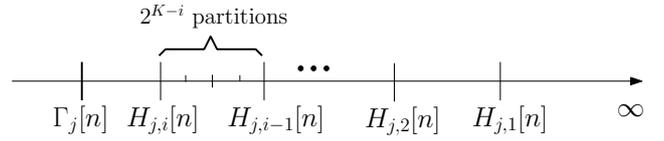


Fig. 5. Query range updated in each minislot.

than one user requests for transmission in the  $i$ -th minislot, the query range will split into two equal-probability partition once a minislot until the SU with highest channel gain is resolved or the remaining  $K - i$  minislots are run out of. For example, as we know the highest channel gain as well as some other SUs' channel gains fall within  $[h_l, h_u)$ , the upper part of the query range  $[h_l, h_u)$ , let say,  $[h_c, h_u)$ , will be chosen as the next query range, where the upper and lower parts of the query range are separated by the threshold  $h_c$  such that

$$h_c = F^{-1}\left(\frac{F(h_u) + F(h_l)}{2}\right). \quad (3)$$

With the newly assigned query range  $[h_c, h_u)$ , suppose that no SUs responds in this moment, *i.e.* no SUs whose channel gains are within the range  $[h_c, h_u)$ . Thereafter, the lower part of the original query range, *i.e.*  $[h_l, h_c)$ , will then be separated into two equal-probability parts and the upper part of  $[h_l, h_c)$  will be chosen as the new query range in the next minislot. Otherwise, if more than one SU responds with the associated query range  $[h_c, h_u)$ , then we separate  $[h_c, h_u)$  and choose the upper part of  $[h_c, h_u)$  again. Hence, clearly, the query range  $[\max\{\Gamma_j[n], H_{j,i}[n]\}, \max\{\Gamma_j[n], H_{j,i-1}[n]\})$  of the first non-idle minislot will be partitioned into  $2^{K-i}$  equal-probability ranges as shown in Fig. 5, and the collision of SUs can be successfully resolved if the SU with highest channel gain falls within one partition among the  $2^{K-i}$  equal-probability ranges *alone*. The average transmission probability of the secondary network in each transmission block will be analyzed in the following subsection.

### C. Average Transmission Probability

Suppose that the lower bound of the query range  $H_{j,i}[n]$  in the  $i$ -th minislot never lower than or equal to  $\Gamma_j[n]$  after  $i - 1$  successive idle minislots, that is,  $F(\Gamma_j[n]) < F(H_{j,i}[n])$ . Since that

$$F(H_{j,i}[n]) = \left(\frac{L-1}{L}\right)^i > 1 - \exp\left(-\frac{\Gamma_j[n]}{\sigma^2}\right),$$

the maximum integer  $i$  such that  $H_{j,i}[n] > \Gamma_j[n]$  is given by

$$\left\lceil \frac{\log\left(1 - \exp\left(-\frac{\Gamma_j[n]}{\sigma^2}\right)\right)}{\log\left(\frac{L-1}{L}\right)} \right\rceil - 1,$$

where  $\lceil x \rceil$  denote the minimum integer that is greater or equal to  $x$ . Let define

$$I_j[n] \triangleq \left\lceil \frac{\log\left(1 - \exp\left(-\frac{\Gamma_j[n]}{\sigma^2}\right)\right)}{\log\left(\frac{L-1}{L}\right)} \right\rceil$$

$$\begin{aligned}
 p_j[n] &= \sum_{i=1}^{\min\{K, I_j[n]-1\}} \sum_{m=1}^{2^{K-i}} \binom{L}{1} \Pr\{\gamma \in [h_{j,i}^{(m-1)}[n], h_{j,i}^{(m)}[n]]\} \left[ \Pr\{\gamma \in [0, h_{j,i}^{(m-1)}[n]]\} \right]^{L-1} \\
 &+ \sum_{m=1}^{2^{K-I_j[n]}} \binom{L}{1} \Pr\{\gamma \in [g_j^{(m-1)}[n], g_j^{(m)}[n]]\} \left[ \Pr\{\gamma \in [0, g_j^{(m-1)}[n]]\} \right]^{L-1} \\
 &= \sum_{i=1}^{\min\{K, I_j[n]-1\}} \sum_{m=1}^{2^{K-i}} L \frac{1}{2^{K-i}} \left[ \frac{1}{L} \left( \frac{L-1}{L} \right)^{i-1} \right] \left[ \left( \frac{L-1}{L} \right)^{i-1} - \frac{2^{K-i} - m + 1}{2^{K-i}} \frac{1}{L} \left( \frac{L-1}{L} \right)^{i-1} \right]^{L-1} \\
 &+ \sum_{m=1}^{2^{K-I_j[n]}} L \frac{\left( \frac{L-1}{L} \right)^{I_j[n]-1} - 1 + \exp\left(-\frac{\Gamma_j[n]}{\sigma^2}\right)}{2^{K-I_j[n]}} \left[ \left( \frac{L-1}{L} \right)^{I_j[n]-1} - \frac{2^{K-I_j[n]} - m + 1}{2^{K-I_j[n]}} \left[ \left( \frac{L-1}{L} \right)^{I_j[n]-1} - 1 + \exp\left(-\frac{\Gamma_j[n]}{\sigma^2}\right) \right] \right]^{L-1}
 \end{aligned} \tag{4}$$

$$\tag{5}$$

as the maximum allowable number of minislots that the lower bound of the query range  $H_{j,i}[n]$  can be successive lowered down. Please note that, the lower bound of the  $I_j[n]$ -th minislot should never be less than  $\Gamma_j[n]$ , the query range of the  $I_j[n]$ -th minislot is given by  $[\Gamma_j[n], H_{j,I_j[n]-1}[n]]$ . The transmission probability in the  $n$ -th transmission block given that  $\mathcal{H}_j$  is detected is given by (4), where  $[h_{j,i}^{(m-1)}[n], h_{j,i}^{(m)}[n]]$  is the  $m$ -th partition of the query range  $[H_{j,i}[n], H_{j,i-1}[n]]$  in the  $i$ -th minislot such that  $h_{j,i}^{(0)}[n] = H_{j,i}[n]$ ,  $h_{j,i}^{(2^{K-i})}[n] = H_{j,i-1}[n]$ , and

$$\Pr\{\gamma \in [h_{j,i}^{(m-1)}[n], h_{j,i}^{(m)}[n]]\} = \frac{\Pr\{\gamma \in [H_{j,i}[n], H_{j,i-1}[n]]\}}{2^{K-i}}.$$

And similarly,  $[g_j^{(m-1)}[n], g_j^{(m)}[n]]$  is the  $m$ -th partition of the query range  $[\Gamma_j[n], H_{j,I_j[n]-1}[n]]$  in the  $I_j[n]$ -th minislot such that  $g_j^{(0)}[n] = \Gamma_j[n]$ ,  $g_j^{(2^{K-I_j[n]})}[n] = H_{j,I_j[n]-1}[n]$ , and

$$\Pr\{\gamma \in [g_j^{(m-1)}[n], g_j^{(m)}[n]]\} = \frac{\Pr\{\gamma \in [\Gamma_j[n], H_{j,I_j[n]-1}[n]]\}}{2^{K-I_j[n]}}.$$

Moreover,

$$\begin{aligned}
 \Pr\{\gamma \in [h_{j,i}^{(m-1)}[n], h_{j,i}^{(m)}[n]]\} &= \frac{F(H_{j,i-1}[n]) - F(H_{j,i}[n])}{2^{K-i}} \\
 &= \frac{\frac{1}{L} \left( \frac{L-1}{L} \right)^{i-1}}{2^{K-i}}
 \end{aligned}$$

denotes the probability that the highest channel gain falls within the  $m$ -th partition and

$$\left[ \Pr\{\gamma \in [0, h_{j,i}^{(m-1)}[n]]\} \right]^{L-1}$$

denotes the probability that the other  $L-1$  SUs' channel gains would not be in the same partition with the highest one, where

$$\begin{aligned}
 &\Pr\{\gamma \in [0, h_{j,i}^{(m-1)}[n]]\} \\
 &= F(H_{j,i-1}[n]) - \Pr\{\gamma \in [h_{j,i}^{(m-1)}[n], H_{j,i-1}[n]]\} \\
 &= \left( \frac{L-1}{L} \right)^{i-1} - \frac{2^{K-i} - m + 1}{2^{K-i}} \frac{1}{L} \left( \frac{L-1}{L} \right)^{i-1}.
 \end{aligned}$$

And the second summation term in (4) can be interpreted in the similar way as above.

However, the derivation of the transmission probability is not easy. We then find an upper bound of the average

transmission probability by the aid of Lemma 1 and use the upper bound as an approximation of the actual average transmission probability in the following throughput analysis. It is shown in section V that the approximation performs well via simulations.

LEMMA 1: Let  $g(m)$  be a decreasing function of parameter  $m$ , then for any finite integer  $M$ , we have

$$\int_0^M g(m) dm \geq \sum_{m=1}^M g(m).$$

*Proof:* The proof is given in Appendix A. Thus, it follows that

$$\begin{aligned}
 &\sum_{m=1}^{2^{K-i}} \left[ 1 - \frac{2^{K-i} - m + 1}{2^{K-i}} \frac{1}{L} \right]^{L-1} = \sum_{m=1}^{2^{K-i}} \left[ 1 - \frac{m}{2^{K-i}} \frac{1}{L} \right]^{L-1} \\
 &\leq \int_{m=1}^{2^{K-i}} \left[ 1 - \frac{1}{2^{K-i}} \frac{m}{L} \right]^{L-1} dm = 2^{K-i} - 2^{K-i} \left[ 1 - \frac{1}{L} \right]^{L-1}
 \end{aligned}$$

and similarly,

$$\begin{aligned}
 &\sum_{m=1}^{2^{K-I_j}} \left[ \left( \frac{L-1}{L} \right)^{I_j-1} - \frac{2^{K-I_j} - m + 1}{2^{K-I_j}} \left[ \left( \frac{L-1}{L} \right)^{I_j-1} - 1 + \exp\left(-\frac{\Gamma_j}{\sigma^2}\right) \right] \right]^{L-1} \\
 &\leq \left( \frac{L-1}{L} \right)^{(I_j-1)L} \frac{2^{K-I_j}/L}{\left( \frac{L-1}{L} \right)^{I_j-1} - 1 + \exp\left(-\frac{\Gamma_j}{\sigma^2}\right)} \left[ 1 - \left( \frac{1 - \exp\left(-\frac{\Gamma_j}{\sigma^2}\right)}{\left( \frac{L-1}{L} \right)^{I_j-1}} \right)^L \right].
 \end{aligned}$$

Hence, the transmission probability given in (5) is approximated with its upper bound as

$$\begin{aligned}
 p_j &\leq \sum_{i=1}^{\min\{K, I_j-1\}} \left[ 1 - \left( 1 - \frac{1}{L} \right)^L \right] \left( \frac{L-1}{L} \right)^{L(i-1)} \\
 &+ \left( \frac{L-1}{L} \right)^{(I_j-1)L} \left[ 1 - \left( \frac{1 - \exp\left(-\frac{\Gamma_j}{\sigma^2}\right)}{\left( \frac{L-1}{L} \right)^{I_j-1}} \right)^L \right] \mathbf{1}_{\{K \geq I_j\}} \\
 &= \min \left\{ 1 - \left( \frac{L-1}{L} \right)^{LK}, 1 - \left( \frac{L-1}{L} \right)^{L(I_j-1)} \right\} \\
 &+ \left( \frac{L-1}{L} \right)^{(I_j-1)L} \left[ 1 - \left( \frac{1 - \exp\left(-\frac{\Gamma_j}{\sigma^2}\right)}{\left( \frac{L-1}{L} \right)^{I_j-1}} \right)^L \right] \mathbf{1}_{\{K \geq I_j\}} \\
 &\triangleq \bar{p}_j,
 \end{aligned} \tag{6}$$

where  $\mathbf{1}_{\{\cdot\}}$  is an indicator function and we denote the upper bound of the average transmission probability  $p_j$  as  $\bar{p}_j$ . Note that the index of minislots  $n$  is omitted here to simplify the expression.

It is worthwhile to note that, the upper bound of the transmission probability given in (6) increases as the threshold  $\Gamma_j$  decreases. However, the transmission probability stops to increase but remains the same whenever  $\Gamma_j \leq H_{j,K}$  (i.e.  $K \leq I_j$ ), where  $H_{j,K}$  is the lower bound of the query range after  $K - 1$  successive idle minislots. Hence, the upper bound of the transmission probability in (6) can be rewritten as

$$\bar{p}_j = 1 - \left[ 1 - \exp\left(-\frac{\max\{\Gamma_j, H_{j,K}\}}{\sigma^2}\right) \right]^L, \quad (7)$$

which is equal to the probability that at least one SU's channel gain is greater than or equal to  $\max\{\Gamma_j, H_{j,K}\}$ .

#### IV. THROUGHPUT ANALYSIS AND OPTIMIZATION

Here, we derive the average throughput for the secondary network as a performance measure of the proposed channel-aware reservation-based scheduling policy.

Under the reservation scheme described in section III, the average throughput of CRN in the  $n$ -th transmission block given that  $\mathcal{H}_j$  is detected in the spectrum sensing period is given by (8), where we assume that the network bandwidth is normalized to 1 and the channel noise is white Gaussian with unit variance. We further assume that the SU who successfully reserves the transmission will transmit with the rate equal to the channel capacity. In fact, the transmission rate is hard to derive since the probability that the transmission is successfully reserved by the SU with the highest channel gain will depend on the joint distribution of all the SUs' channel gains. We then derive an upper bound of the transmission rate, which is denoted as  $\bar{R}_j[n]$  in the  $n$ -th transmission block given that  $\mathcal{H}_j$  is detected. Intuitively, the transmission probability  $p_j[n]$  is upper bounded by the probability that at least one SU's channel gain is greater than or equal to  $\max\{\Gamma_j, H_{j,K}\}$ . Hence, the upper bound of the transmission rate is defined as the average channel capacity if the SU with the highest channel gain is always served at each transmission block provided that the highest channel gain is greater than or equal to  $\max\{\Gamma_j, H_{j,K}\}$ . More specifically, the transmission rate is upper bounded as

$$R_j \leq \int_{\max\{\Gamma_j, H_{j,K}\}}^{\infty} \log(1 + \gamma_{max}) f_{max}(\gamma_{max}) d\gamma_{max} \triangleq \bar{R}_j \quad (9)$$

where  $\gamma_{max} \triangleq \max_i \gamma_i$  is the highest channel gain among all SUs, and

$$f_{max}(\gamma_{max}) = \frac{L}{\sigma^2} \exp\left(-\frac{\gamma_{max}}{\sigma^2}\right) \left[1 - \exp\left(-\frac{\gamma_{max}}{\sigma^2}\right)\right]^{L-1}$$

is the density function of the random variable  $\gamma_{max}$ . The transmission block index  $n$  is also omitted here to simplify the expression in the above equations.

Let define

$$T = T_S + N \cdot T_X = T_S + N(T_C + T_R + T_D + T_{ACK})$$

as the sensing interval from one spectrum sensing period to the next spectrum sensing period, where the variable  $T_S$  indicates the time duration for the secondary network to perform the spectrum sensing. The average throughput of the secondary network given the thresholds  $\{(\Gamma_0[n], \Gamma_1[n])\}_{n=1}^N$  on the SUs' channel gains, which is defined as the average transmission data amount in one sensing interval over the period of one sensing interval, can be given by

$$\begin{aligned} & U(\{(\Gamma_0[n], \Gamma_1[n])\}_{n=1}^N) \\ &= \frac{1}{T} \sum_{i=0}^1 v_i b_i[n] \sum_{j=0}^1 \sum_{n=1}^N \Pr\{\mathcal{H}_j \text{ detected} | \mathcal{H}_i \text{ true}\} R_j[n] T_D \\ &= \frac{T_D}{T} \sum_{n=1}^N \left\{ R_0[n] \{v_0(1 - P_{FA})b_0[n] + v_1 P_M b_1[n]\} \right. \\ & \quad \left. + R_0[n] \{v_0 P_{FA} b_0[n] + v_1(1 - P_M) b_1[n]\} \right\}, \quad (10) \end{aligned}$$

where  $b_j[n]$  is the probability that the PU network remains idle in the  $n$ -th transmission block given that  $\mathcal{H}_j$  is true in the spectrum sensing period<sup>1</sup>. In order to provide sufficient protection to the PU signal, the optimal thresholds of channel gain  $\Gamma_0^*[n], \Gamma_1^*[n]$  are found by maximizing the throughput of the secondary network while limiting the collision probability as given in (1), i.e.

$$\begin{aligned} & \{(\Gamma_0^*[n], \Gamma_1^*[n])\}_{n=1}^N = \arg \max_{\{(\Gamma_0[n], \Gamma_1[n])\}_{n=1}^N} U(\{(\Gamma_0[n], \Gamma_1[n])\}_{n=1}^N) \\ & \text{subject to } \frac{p_0[n]c_0[n] + p_1[n]c_1[n]}{v_0(1 - b_0[n]) + v_1(1 - b_1[n])} \leq p_c, \\ & \quad n = 1, 2, \dots, N. \quad (11) \end{aligned}$$

Obviously, the optimization can be done transmission block by transmission block since that collision probability constraint is also given block by block. However, since that the transmission probability and rate are hard to derive. Here, we replace the transmission probability and rate with the upper bound of those as given in (7) and (9) into (11). That is, the suboptimal thresholds  $\{(\Gamma_0^\dagger[n], \Gamma_1^\dagger[n])\}_{n=1}^N$  can be found by

$$\begin{aligned} & (\Gamma_0^\dagger[n], \Gamma_1^\dagger[n]) = \arg \max_{(\Gamma_0[n], \Gamma_1[n])} \bar{U}[n] \\ & \text{subject to } \frac{\bar{p}_0[n]c_0[n] + \bar{p}_1[n]c_1[n]}{v_0(1 - b_0[n]) + v_1(1 - b_1[n])} \leq p_c, \quad (12) \end{aligned}$$

where

$$\bar{U}[n] \triangleq \bar{R}_0[n]a_0[n] + \bar{R}_1[n]a_1[n]$$

is the upper bound of the average transmission rate in the  $n$ -th transmission block. The variable  $a_0[n]$  and  $a_1[n]$  are defined as

$$a_0[n] \triangleq v_0(1 - P_{FA})b_0[n] + v_1 P_M b_1[n]$$

<sup>1</sup>Please note that,  $b_i[n]$  denotes the probability that the PU network remains idle in the *whole transmission block*  $n$ . Suppose that the SU network transmits in one transmission block, a collision occurs even if the PU network is just active for a short period of time in that transmission block. Moreover, no matter when the PU is active during that transmission block, the transmission of the SU signal will fail.

$$R_j[n] = \sum_{i=1}^{\min\{K, I_j[n]-1\}} \sum_{m=1}^{2^{K-i}} \binom{L}{1} \int_{h_{j,i}^{(m-1)}[n]}^{h_{j,i}^{(m)}[n]} \log(1+\gamma) f(\gamma) d\gamma \left[ \Pr \left\{ \gamma \in [0, h_{j,i}^{(m-1)}[n]] \right\} \right]^{L-1} + \sum_{m=1}^{2^{K-I_j[n]}} \binom{L}{1} \int_{g_j^{(m-1)}[n]}^{g_j^{(m)}[n]} \log(1+\gamma) f(\gamma) d\gamma \left[ \Pr \left\{ \gamma \in [0, g_j^{(m-1)}[n]] \right\} \right]^{L-1} \quad (8)$$

and

$$a_1[n] \triangleq v_0 P_{FA} b_0[n] + v_1 (1 - P_M) b_1[n].$$

Since that the optimization parameter  $\Gamma_j[n]$  has a one-to-one mapping to the upper bound of the probability  $\bar{p}_j[n]$  given that  $\Gamma_j[n] \geq H_{j,K}[n]$ , that is,

$$\Gamma_j[n] = -\sigma^2 \log \left( 1 - (1 - \bar{p}_j[n])^{(1/L)} \right),$$

which is followed from (7). Furthermore, the rate  $\bar{R}_j[n]$  in (9) is an increasing and concave function of the transmission probability's upper bound  $\bar{p}_j[n]$ . Hence, the optimal transmission probabilities' upper bounds in each transmission block are given by

$$(\bar{p}_0^\dagger[n], \bar{p}_1^\dagger[n]) = \arg \max_{(\bar{p}_0[n], \bar{p}_1[n])} \bar{U}[n] \quad (13)$$

$$\begin{aligned} \text{subject to } & \frac{\bar{p}_0[n]c_0[n] + \bar{p}_1[n]c_1[n]}{v_0(1 - b_0[n]) + v_1(1 - b_1[n])} \leq p_c, \\ & 0 \leq \bar{p}_0[n] \leq 1 - [F(H_{0,K}[n])]^L, \\ & 0 \leq \bar{p}_1[n] \leq 1 - [F(H_{1,K}[n])]^L, \end{aligned}$$

which can be solved by any convex optimization procedures. Hence, the suboptimal threshold in the  $n$ -th transmission block given that  $\mathcal{H}_j$  detected is given by

$$\Gamma_j^\dagger[n] = -\sigma^2 \log \left( 1 - (1 - \bar{p}_j^\dagger[n])^{(1/L)} \right).$$

## V. SIMULATIONS

In this section, we show the network throughput for the secondary network under the proposed channel-aware reservation-based MAC protocol via simulations. We also compare the proposed MAC protocol with the random polling protocol without channel awareness. In the later case, given that the spectrum state  $\mathcal{H}_j$  is detected, the AP will randomly polls one SU to transmit in the  $n$ -th transmission block with probability  $\hat{p}_j[n]$ , where the transmission probabilities  $(\hat{p}_0[n], \hat{p}_1[n])$  meet the collision constraint

$$\frac{\hat{p}_0[n]c_0[n] + \hat{p}_1[n]c_1[n]}{v_0(1 - b_0[n]) + v_1(1 - b_1[n])} \leq p_c$$

and then maximize the average transmission rate in the  $n$ -th transmission block, *i.e.*

$$\int_0^\infty \log(1+\gamma) f(\gamma) d\gamma \{ \hat{p}_0[n]a_0[n] + \hat{p}_1[n]a_1[n] \}$$

as well. Moreover, to examine the effectiveness of replacing the average transmission probability and average transmission rate with the corresponding upper bounds in the maximization problem (12), the throughput in (10) is depicted with the

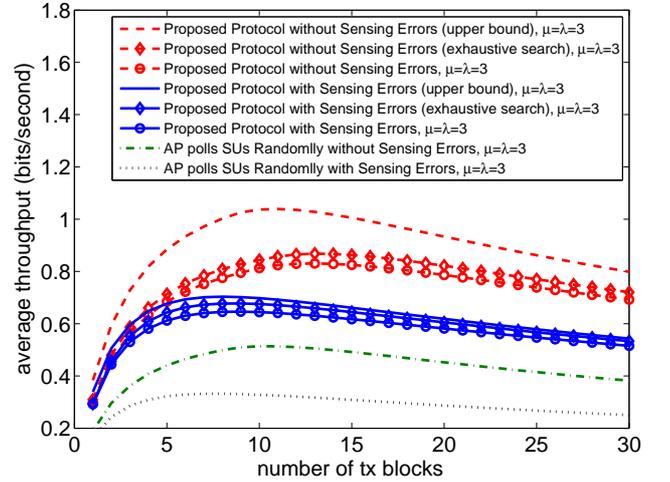
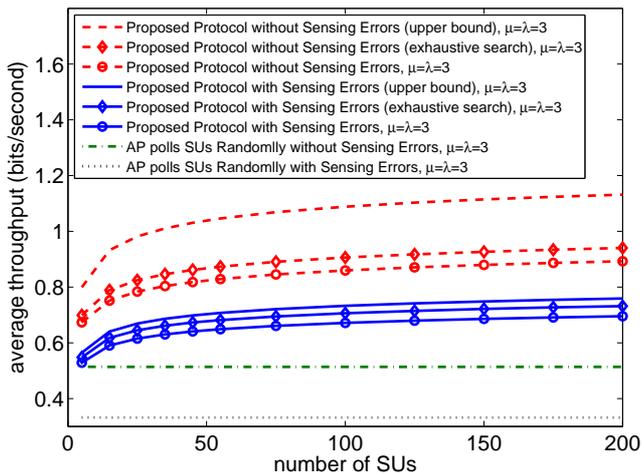
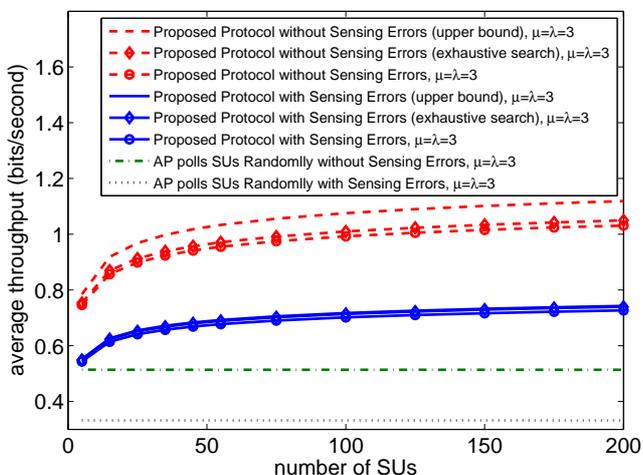


Fig. 6. Network throughput versus the number of transmission blocks.

optimal solutions found in (11) and the suboptimal solutions in (13), respectively. The optimal solutions are derived via the exhaustive searching and the corresponding throughput is labeled with *exhaustive search* (ES). In addition, the upper bound of the throughput, *i.e.*  $\sum_{n=1}^N \frac{T_D}{T} \bar{U}[n]$  is labeled with *upper bound* (UB) in our simulations.

Throughout our simulations, the spectrum sensing error (SSE) is characterized with the false-alarm probability  $P_{FA} = 0.1$  and the miss detection probability  $P_M = 0.05$ . Also, we set  $P_{FA} = P_M = 0$  to indicate the case without spectrum sensing error. The proposed reservation-based scheme is operated with the following parameters:  $T_X = 2000 \times 10^{-6}$  second,  $T_C = T_{min} = T_{ACK} = 20 \times 10^{-6}$  second, and  $T_S = 5000 \times 10^{-6}$  second. Suppose that the collision probability is upper bounded with  $p_c = 5\%$  for both the proposed scheme and the random polling scheme. The channel between SUs to AP are distributed as *i.i.d.* complex Gaussian random variables with zero mean and variance  $\sigma^2 = 4$  in each transmission block. Consider that the secondary network with  $L = 50$  SUs and the PU signals change with rate  $\mu = \lambda = 3$ . Fig. 6 shows the throughput of the secondary network by assuming that there are  $K = 4$  minislots in each transmission block. Note that, in the random polling protocol, each SU transmits immediately whenever it is polled by the AP. No minislots is needed for SUs to reserve the channel in the random polling protocol. The tradeoff between the number of data transmission blocks and the network throughput is observed in Fig. 6. Intuitively, while the number of data transmission blocks  $N$  is large, SUs spend most of the time


 Fig. 7. Network throughput versus the number of SUs with  $K = 4$ .

 Fig. 8. Network throughput versus the number of SUs with  $K = 6$ .

transmitting. The network throughput is supposed to increase with  $N$ . However, since that the spectrum sensing outcome becomes unreliable as time goes by, the secondary network is forced to monitor the spectrum occupancy state with a suitable spectrum sensing interval such that the throughput degradation due to the uncertainty of the spectrum occupancy state or due to the sparse data transmission time can be eliminated as much as possible.

Fig. 7 and 8 show the relationship between the network throughput and the number of SUs with  $K = 4$  and 6 minislots respectively, where the optimal transmission block number is summarized in Table I and II. Clearly, with more minislots (with larger  $K$ ), the higher probability that the SU with the highest channel gain can be resolved would be. Hence, the upper bounds of the transmission probability and rate given by assuming that the collision among SUs can be resolved no matter how many minislots are allocated are almost equal to the actual transmission probability and rate indeed. So we can see from the simulations that the throughput

 TABLE I  
OPTIMAL NUMBER OF TRANSMISSION BLOCK  $N$  IN FIG. 7.

Transmission scheme	$L = 5$	$L = 25$	$L = 55$	$L = 100$
Proposed w/o SSE (ES)	13	13	13	13
Proposed w/o SSE	11	11	11	11
Proposed w/ SSE (ES)	9	9	9	9
Proposed w/ SSE	9	8	8	8
Random Polling w/o SSE	10	10	10	10
Random Polling w/ SSE	8	8	8	8

 TABLE II  
OPTIMAL NUMBER OF TRANSMISSION BLOCK  $N$  IN FIG. 8.

Transmission scheme	$L = 5$	$L = 25$	$L = 55$	$L = 100$
Proposed w/o SSE (ES)	12	12	11	11
Proposed w/o SSE	11	11	11	11
Proposed w/ SSE (ES)	9	8	8	8
Proposed w/ SSE	9	8	8	8
Random Polling w/o SSE	10	10	10	10
Random Polling w/ SSE	8	8	8	8

under the suboptimal transmission strategy almost coincides with the average throughput under the optimal transmission strategy found by exhaustive search with a sufficient large  $K$ . Moreover, the number of minislots required to resolve the collision does not need to be very large, for example,  $K = 6$  performs very well in our simulations. On the other hand, it can be seen that the average throughput under the proposed channel-aware MAC protocol increases with the number of secondary users, which interprets for the multiuser diversity. On the contrary, the throughput under the random polling policy remains the same no matter how the network size grows and is much less than the throughput under the proposed MAC protocol.

## VI. CONCLUSIONS

We proposed a channel-aware reservation-based MAC protocol for the secondary networks where the transmission of secondary users takes into consideration the local CSI at each SU as well as the spectrum sensing reliability in each transmission time block. A collision probability constraint was imposed on the secondary network in order to provide sufficient protection for the primary network. The average throughput of the secondary network was studied in this work and a tradeoff could then be observed between the amount of resources allocated to spectrum sensing and actual data transmission via simulations.

## APPENDIX A PROOF OF LEMMA 1

Suppose that  $g(m)$  is a decreasing function with  $m$ . It is true that

$$\begin{aligned} \int_0^M g(m)dm &= \sum_{m'=1}^M \int_{m'-1}^{m'} g(m)dm \\ &\stackrel{(a)}{\leq} \sum_{m'=1}^M \int_{m'-1}^{m'} g(m')dm = \sum_{m'=1}^M g(m'), \end{aligned}$$

where (a) applies since that  $g(m)$  is a decreasing function of  $m$  and  $g(m) \geq g(m')$  for all  $m' \geq m$ . Hence, we complete the proof.

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