All Pass Filtered Reference LMS Algorithm for Adaptive Feedback Active Noise Control

Iman Tabatabaei Ardekani, Waleed H. Abdulla and Sing-Kiong Nguang
The University of Auckland, Auckland, New Zealand
E-mail: itab001@aucklanduni.ac.nz, w.abdulla@auckland.ac.nz, sk.nguang@auckland.ac.nz

Abstract— **This paper presents a new structure for adaptive feedback active noise control. It is shown that the minimum phase component of the secondary path can be directly compensated while ANC controller generates the anti-noise signal. For compensation of the all pass component, the reference signal is filtered by only the all pass component of the secondary path and, then, used in the least mean square adaptation algorithm. Simulation results show that the proposed structure can dramatically increase the convergence rate of the adaptive feedback active noise controllers and also improve the band width within which adaptive feedback active noise control is efficient.**

I. Introduction

Although passive noise controllers made by sound absorbing materials are valued for high attenuation over a broad frequency range, they are inappropriate for many modern applications. It is mainly because they are relatively bulky, costly, and ineffective for low frequencies. To overcome these problems, the theory of Active Noise Control (ANC) was proposed by Paul Leug in 1930's [1]. Leug suggested an electro-acoustical apparatus to reproduce sound of equal magnitude but opposite phase for cancellation of tonal sound in an acoustic duct. Leug was unable to realize his idea because of the technological limitations. Two decade later (in 1950's), Olsen applied analog electronic technology to invent the first realization of ANC, called "Electronic Sound Absorber" [2]. He also showed potential capabilities of the ANC for hearing enhancement in cars, airplanes and industrial work places. Olsen's invention and other early realizations of ANC had an analog amplifier which coupled a loudspeaker to a microphone by a simple negative feedback. The common drawback of these systems was that they are non-adaptive in nature.

With the advent of the digital technology, realization of the ANC systems was revolutionized. Actually, what is known today as the ANC is so far from imagination of pioneers in the field. It is because today ANC controllers are implemented using adaptive digital filters instead of analog amplifiers. The idea of adaptive ANC was developed by Widrow in 1975 [3]. Widrow showed successful applications of adaptive ANC in electrocardiography, telecommunication and speech enhancement. After that, several researchers began to apply adaptive ANC for acoustic noise. The first report on successful application of adaptive ANC for acoustic noise was published by Burgess in 1981 [4]. Finally in 1984, 50 years after that Leug patented the idea of ANC, Warnaka implemented and patented the first adaptive ANC system in an acoustic duct [5].

There are two different approaches to adaptive ANC: adaptive feedforward and adaptive feedback. In adaptive feedforward ANC, one or more reference microphone(s) is placed outside the desired quiet zone to identify the noise field [3], [4]. However, in some applications using a number of microphones is not technically or economically possible. To overcome this problem, adaptive feedback ANC was suggested by Eriksson in 1991 [6]. In this approach, the noise field is identified using a feedback estimator therefore there is no need to apply any reference microphone. Unfortunately, performance of the adaptive feedback ANC is not as high as adaptive feedforward ANC. Moreover, the band width within which feedback ANC is efficient is very restricted [7]. This paper develops a new approach to overcome available weaknesses of the adaptive feedback ANC.

The rest of this paper is organized as follows. Section II represents mathematical description of the adaptive feedback ANC. Section III describes the proposed structure. Section IV details the computer simulation results and Section V gives some concluding remarks.

II. Adaptive Feedback ANC

When ANC system operates, the electrical anti-noise signal calculated by the ANC controller is converted to an acoustic signal by a cancelling loudspeaker and travels through the air to reach to the desired quiet zone. In this case, measurable noise at the desired quiet zone can be expressed as follows:

$$e(n) = d(n) - s(n) * y(n) \tag{1}$$

where $*$ denotes linear convolution, n is discrete time, $d(n)$ is environmental noise at the quiet zone, $y(n)$ is electrical anti-noise signal generated by the ANC controller and $s(n)$ is impulse response of the signal path from output of the controller to the quiet zone. In ANC literature, $e(n)$ is called residual noise, $d(n)$ is called primary noise and the system whose impulse response is $s(n)$ is called secondary path. Equation (1) describes the acoustic plant in the ANC problem. According to this equation, acoustic primary noise $d(n)$ and electrical anti-noise $y(n)$ are input signals of the acoustic plant and acoustic residual noise $e(n)$ is its output signal.

In ANC systems, a microphone called error microphone is usually used to convert the residual noise signal into electric domain. The optimal value of anti-noise signal $y(n)$ causes that residual noise becomes zero. Therefore, setting (1) into zero leads to obtain the optimal anti-noise:

$$y_{opt}(n) = s^{-1}(n) * d(n) \tag{2}$$

where $y_{opt}(n)$ denotes optimal anti-noise at time n and $s^{-1}(n)$ is impulse response of the secondary path inverse system. Therefore, according to (2), calculation of the optimal anti-noise signal involves two main problems: identification of the primary noise and compensation of the secondary path.

A. *Feedback Estimator for Primary Noise Identification*

From (1), it can be concluded that if $\hat{s}(n)$ is an estimate of the secondary path impulse response then $d(n)$ can be estimated as follows:

$$\hat{d}(n) = e(n) + \hat{s}(n) * y(n) \tag{3}$$

This equation describes the feedback estimator in the adaptive feedback ANC. Now that $\hat{d}(n)$ as an estimate of the primary noise is available, (2) can be approximated as follows:

$$y_{opt}(n) \approx s^{-1}(n) * \hat{d}(n) \tag{4}$$

B. *Adaptive Controller for Secondary Path Compensation*

According to (4), if $\hat{d}(n)$ is available the optimal anti-noise signal can be obtained by estimating the secondary path inverse system. In general case, secondary path can have a number of zeros at the origin or outside the unit circle; therefore its inverse system is not necessarily causal and stable. In this case, according to (4), implementation of $y_{opt}(n)$ is not physically possible. To overcome this problem, an adaptation algorithm can be applied in order to make a causal and stable approximation of $s^{-1}(n)$. If $w_{opt}(n)$ denotes a causal and stable approximation of $s^{-1}(n)$ then (4) can be re-expressed as follows:

$$y_{opt}(n) \approx w_{opt}(n) * \hat{d}(n) \tag{5}$$

$$w_{opt}(n) \approx s^{-1}(n) \tag{6}$$

The optimal ANC controller can be implemented using (5). Therefore, if an adaptation algorithm adjusts impulse response of the ANC controller to $w_{opt}(n)$ then this controller can produce the optimal anti-noise signal in response of $\hat{d}(n)$. In general case, if $w(n)$ is impulse response of the ANC controller, anti-noise signal $y(n)$ can be expressed as follows:

$$y(n) = w(n) * \hat{d}(n) \tag{7}$$

$w(n)$ can be modeled by a adaptive FIR filter with coefficients of $w_0(n), w_1(n), \dots, w_{L-1}(n)$:

$$w(n) = \sum_{k=0}^{L-1} w_k(n).\delta(n-k) \tag{8}$$

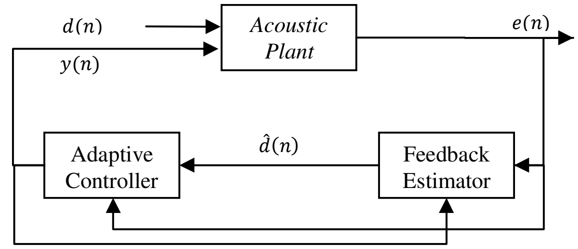

Fig.1 : General block diagram of adaptive feedback ANC

where δ is Kronecker delta function $w_k(n)$ is k-th coefficient of the filter and L is the number of coefficients. It can be shown that if coefficients are updated as follows [7]:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu\acute{\mathbf{x}}(n)e(n) \tag{9}$$

$$\mathbf{w}(n) = [w_0(n) \quad w_1(n) \quad \dots \quad w_{L-1}(n)]^T$$

$$\acute{\mathbf{x}}(n) = [\acute{x}(n) \quad \acute{x}(n-1) \quad \dots \quad \acute{x}(n-L+1)]^T$$

then $w(n)$ converges to $w_{opt}(n)$. In (9), scalar parameter μ is called adaptation step size and $\acute{x}(n)$ represents filtered reference signal:

$$\acute{x}(n) = \hat{s}(n) * \hat{d}(n) \tag{10}$$

Here $\hat{s}(n)$ is again an estimate of the secondary path. This adaptation algorithm is called Filtered-x Least Mean Square (FxLMS) adaptation algorithm. One drawback of the application of this algorithm in adaptive feedback ANC is that it is only efficient for narrow-band noise. It is because when the FxLMS is applied to approximate a non-causal optimal system, its performance is highly depended on the predictability of the noise [7]. Therefore, in adaptive feedback ANC where the optimal ANC controller is non-causal the FxLMS algorithm can only be efficient for predictable noise such as narrow-band noise signals.

Fig. 1 shows general block diagram of the adaptive feedback ANC and the relation between its major parts. As shown in this figure, acoustic plant produces the residual error $e(n)$ in response of the primary noise $d(n)$ and anti-noise $y(n)$. Equation (1) shows the relation between input and output signals of the acoustic plant. Feedback estimator calculates $\hat{d}(n)$ as an estimate of the primary noise signal, $d(n)$. This estimation is formulated by (3). Adaptive controller produces the anti-noise signal $y(n)$in response of the $\hat{d}(n)$. This process is formulated by equation (7). Coefficients of the ANC controller are updated using the adaptation algorithm described by (9) and (10). According to these equations, adaptation algorithm requires residual error and an estimate of the primary noise to update coefficients of the ANC controller.

From the above discussion, adaptive feedback ANC can be implemented using (1), (3), (7), (9) and (10). This implementation is shown in Fig. 2. As shown in this figure, both feedback estimator and adaptive controller require an estimate of the secondary path. Therefore, an essential part of the adaptive feedback ANC is a system for secondary path modeling. Several techniques [8], [9], [10] have been so far proposed for this purpose. This paper focuses on a new adaptive feedback ANC algorithm. Similar to other ANC algorithms; it is assumed that an accurate estimate of the secondary path is available for the control system.

III. Proposed Adaptive Feedback ANC System

Assume that $s(n)$ is an all-zero system. In this case, if $s_M(n)$ is minimum phase component and $s_A(n)$ is all pass component of $s(n)$ then:

$$s(n) = s_M(n) * s_A(n) \tag{11}$$

The minimum phase component $s_M(n)$ is a FIR system whose zeros are located inside the unit circle and the all pass component $s_A(n)$ is an IIR system that passes all frequencies equally, but changes the phase relationship between various frequencies. Therefore amplitude of the all pass component frequency response is 1 for all frequencies. Substituting (11) into (6) results in:

$$w_{opt}(n) = s_M^{-1}(n) * s_A^{-1}(n) \tag{12}$$

$s_M(n)$ is minimum phase component therefore $s_M^{-1}(n)$ is causal and stable but $s_A(n)$ is non-minimum phase therefore its inverse system does not exist. Assuming that $a_{opt}(n)$ is a causal and stable approximation of $s_A^{-1}(n)$, (12) can be re-expressed as follows:

$$w_{opt}(n) \approx s_M^{-1}(n) * a_{opt}(n) \tag{13}$$

For implementation of the first term of (13), an estimate of the $s_M(n)$ is required. The second term can be implemented using adaptive filter $a(n)$. Therefore, impulse response of the ANC controller can be approximated as:

$$w(n) \approx \hat{s}_M^{-1}(n) * a(n) \tag{14}$$

where $\hat{s}_M(n)$ is an estimate of $s_M(n)$. Impulse response $a(n)$ can be modeled using a finite number of coefficients:

$$a(n) = \sum_{k=0}^{L-1} a_k(n).\delta(n-k) \tag{15}$$

Coefficients $a_0(n), a_1(n), \ldots, a_{L-1}(n)$ should be obtained using an adaptation algorithm. This algorithm should be updated in such a way that $a(n)$ converges to its optimal value, $a_{opt}(n)$. This algorithm which we call it ***All Pass FxLMS*** is derived as follows. Substituting (14) into (7), anti-noise signal can be obtained:

$$y(n) = s_M^{-1}(n) * a(n) * \hat{d}(n) \tag{16}$$

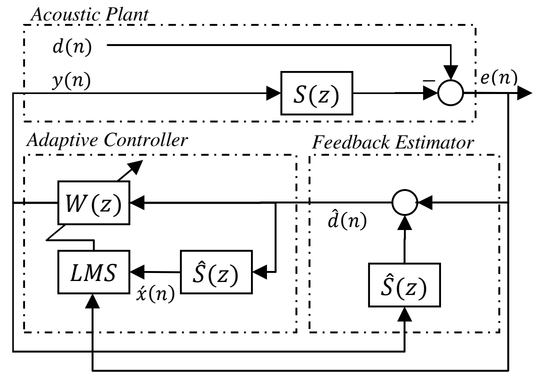

Fig. 2: Available implementation of adaptive feedback ANC

From (1) and (16), it can be concluded that:

$$e(n) = d(n) - s(n) * s_M^{-1}(n) * a(n) * \hat{d}(n) \tag{17}$$

Using (11), this equation can be simplified as follows:

$$e(n) = d(n) - s_A(n) * a(n) * \hat{d}(n) \tag{18}$$

This equation can be rewritten as follows:

$$e(n) = d(n) - a(n) * \acute{x}_A(n) \tag{19}$$

where $\acute{x}_A(n)$ is all pass filtered primary noise signal:

$$\acute{x}_A(n) = s_A(n) * \hat{d}(n) \tag{20}$$

If error function $\xi(n)$ is defined as the mean squared residual noise signal then according to the concept of steepest descent the following recursive equation minimizes $\xi(n)$ [7]:

$$\mathbf{a}(n+1) = \mathbf{a}(n) - \frac{\mu}{2}\nabla_{\mathrm{a}}\xi(n) \tag{21}$$

where scalar parameter μ is adaptation step size and coefficient vector $\mathbf{a}(n)$ is defined as follows:

$$\mathbf{a}(n) = [a_0(n) \quad a_1(n) \quad \ldots \quad a_{L-1}(n)]^T \tag{22}$$

and $\nabla_{\mathrm{a}}\xi(n)$ denotes the gradient of error function with respect to $\mathbf{a}(n)$:

$$\nabla_{\mathrm{a}}\xi(n) = 2e(n)\nabla_{\mathrm{a}}e(n) = \tag{23}$$

$$2e(n)\left[\frac{\partial e(n)}{\partial a_0(n)} \quad \frac{\partial e(n)}{\partial a_1(n)} \quad \ldots \quad \frac{\partial e(n)}{\partial a_{L-1}(n)}\right]^T$$

Substituting (15) into (19) and the result into (23) leads to the following result:

$$\nabla_{\mathrm{a}}\xi(n) = -2\acute{\mathbf{x}}_A(n)e(n) \tag{24}$$

where:

$$\acute{\mathbf{x}}_A(n) = [\acute{x}_A(n) \quad \acute{x}_A(n-1) \quad \ldots \quad \acute{x}_A(n-L+1)]^T \qquad (25)$$

Substituting (24) into (21) leads to:

$$\mathbf{a}(n+1) = \mathbf{a}(n) + \mu \acute{\mathbf{x}}_A(n) e(n) \qquad (26)$$

This updating equation is similar to the updating equation of the FxLMS algorithm. The only difference is that, in FxLMS algorithm filtered reference signal is calculated by filtering $\hat{d}(n)$ through an estimate of the secondary path but in the proposed algorithm this signal is filtered through only all pass component of the secondary path. In practice all pass filtered reference signal $\acute{x}_A(n)$ is not physically available therefore it should be estimated as follows:

$$\acute{x}_A(n) \approx \hat{s}_A(n) * \hat{d}(n) \qquad (27)$$

where $\hat{s}_A(n)$ is an available estimate of $s_A(n)$. Fig. 3 shows implementation of the proposed structure for the adaptive feedback ANC system. As shown by this figure, the ANC controller is implemented using (14). According to this equation, the ANC controller is a series connection of $\hat{s}_M^{-1}(n)$ and adaptive filter $a(n)$. This adaptive filter can be implemented using equation (16) and its adaptation algorithm is described by equations (22) and (25)-(27).

Implementation of the traditional ANC controllers requires an offline or online estimate of the secondary path. According to Fig. 3, implementation of the proposed controller requires decomposition of the estimated secondary path into an all pass and a minimum phase component. However, if an offline estimate of the secondary path is available, it can be composed into its all pass and minimum phase components using traditional methods described in available literature [11].

IV. SIMULATION RESULTS

Effectively of the proposed ANC algorithm has been verified using several computer simulations. Specification of the secondary path is taken from the floppy disc attached with [7]. The original secondary path, $s(n)$, is modeled by an ARMA process and its estimation, $\hat{s}(n)$, is modeled by a FIR filter of order 24. Fig. 4 shows frequency response of the original and estimated secondary paths. Fig. 5 shows location of zeros of the estimated secondary path in z-plane. According to (6), these zeros are identical to the poles of the optimal feedback ANC controller. As shown by this figure, the optimal ANC controller is not stable because it has two poles outside the unit circle. However, as discussed before, adaptation algorithm tries to make a stable approximation of the optimal controller. The estimated secondary path is decomposed into a FIR minimum phase component and an IIR all-pass component. Order of the minimum phase component is set to 22 (equal to the number of stable zeros of the secondary path) and the all pass component has two zeros and two poles (equal to the number of unstable zeros of the secondary path). Using this secondary path estimation and decomposition, three simulation experiments are discussed in the following.

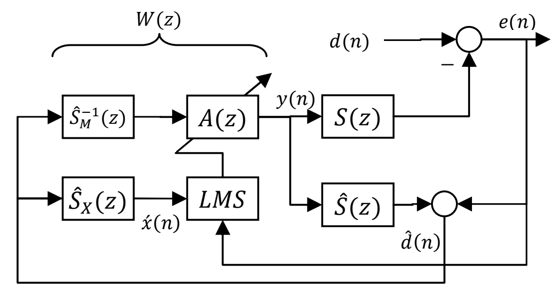

Fig. 3: The proposed structure for adaptive feedback ANC

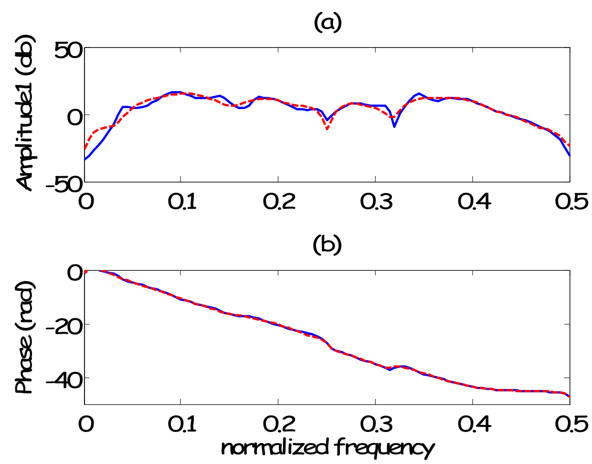

Fig. 4: Frequency response of the secondary path. a) solid: frequency response amplitude of the real system, dashed: frequency response amplitude of the estimated system. b) solid: frequency response phase of the real system, dashed: frequency response phases of the estimated system

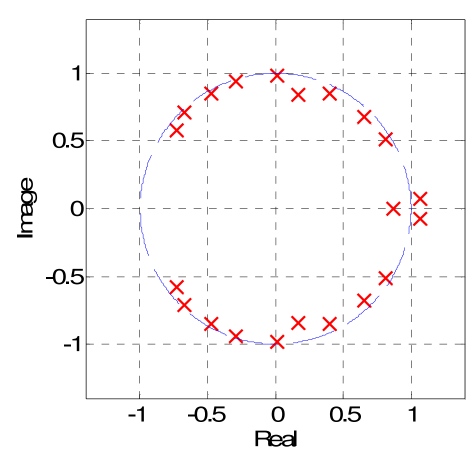

Fig. 5: Root locus of the optimal feedback ANC controller

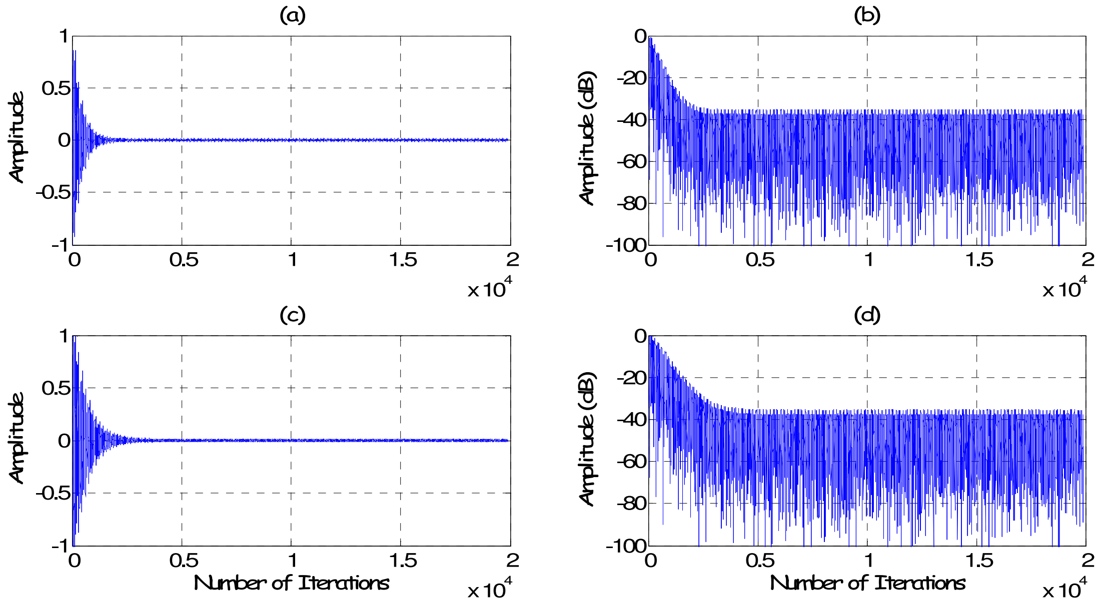

Fig. 6: Simulation results in case 1. a) residual noise obtained from the proposed algorithm b) residual noise power in db obtained from the proposed algorithm c) residual noise obtained from the FxLMS algorithm d) residual noise power in db obtained from the FxLMS algorithm

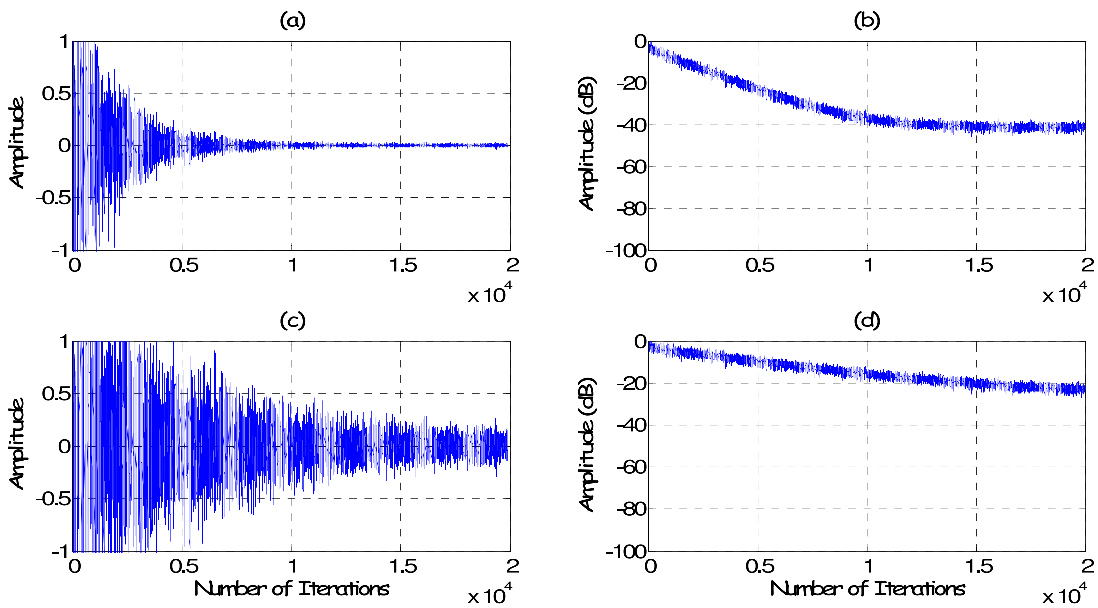

Fig. 7: Simulation results in case 2. a) residual noise obtained from the proposed algorithm b) MSE in db obtained from the proposed algorithm c) residual noise obtained from the FxLMS algorithm d) MSE in db obtained from the FxLMS algorithm

A. Case 1

In this case primary noise is a narrow-band signal comprising frequencies of 100, 175, 200, and 225 Hz. The variance of the noise is adjusted to 1. The sampling rate is 4 KHz. The simulation results including the residual noise in time domain and its power are shown in Fig. 6. It can be seen that the proposed algorithm can attenuate the narrow-band noise at a faster convergence rate as compared to the FxLMS algorithm. Measurement on the simulation results shows that the proposed algorithm converges after about 190 iterations but the FxLMS algorithm converges after 280 iterations. However, in this case FxLMS has acceptable performance.

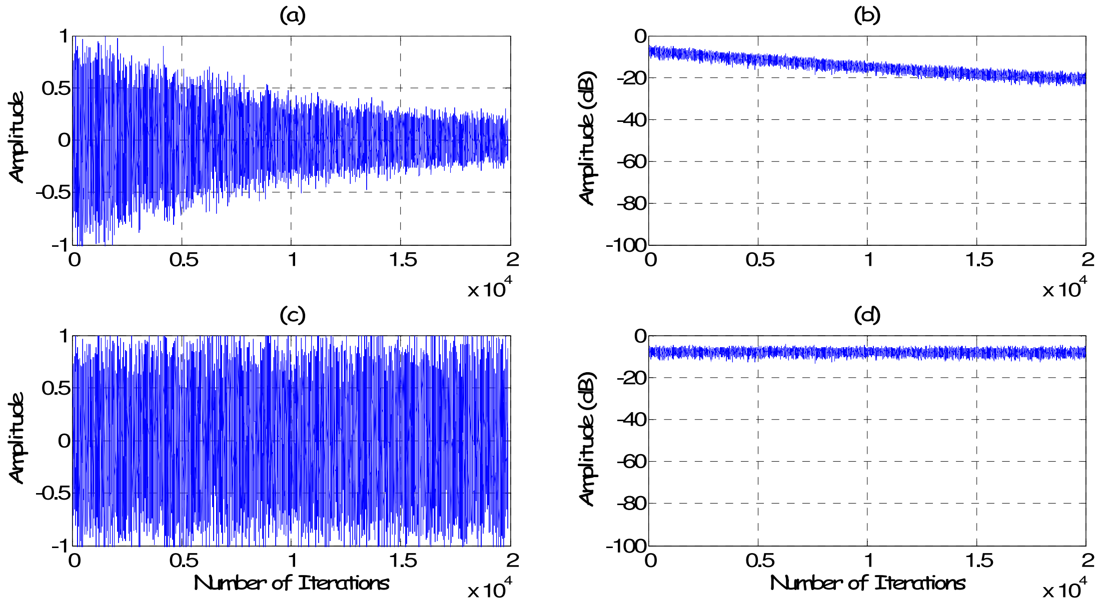

Fig. 8: Simulation results in case 3. a) residual noise obtained from the proposed algorithm b) MSE in db obtained from the proposed algorithm c) residual noise obtained from the FxLMS algorithm d) MSE in db obtained from the FxLMS algorithm

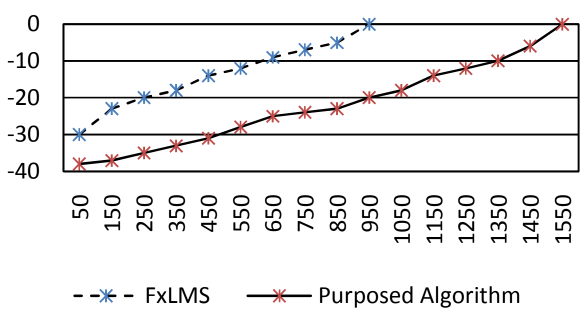

Fig. 9: Attenuation of the proposed algorithm (solid line) and the FxLMS (dashed line) with different noise band width

B. Case 2

In this case primary noise is a broad-band noise which is obtained by filtering a white Gaussian noise of unit variance through a bandpass filter with the pass band 100–250 Hz. The simulation results are shown in Fig. 7. It can be seen that both the proposed algorithm and the FxLMS algorithm can attenuate the noise but the proposed algorithm is faster and results in lower residual noise. The proposed algorithm converges after about 14000 iterations but the FxLMS algorithm converges after about 20000 iterations. The minimum residual noise of the proposed algorithm is about -38 dB but that of FxLMS algorithm is about -23 dB.

C. Case 3

In this case primary noise is a broad-band noise which is obtained filtering a white Gaussian noise of unit variance through a bandpass filter with the pass band 100–1000 Hz. The simulation results are shown in Fig. 8. It can be seen that the FxLMS algorithm cannot attenuate this broad-band noise but the proposed algorithm converges after about 20000 iterations. The residual noise obtained by the proposed algorithm is about -20 dB.

The above three simulation results show that the proposed algorithm can improve the band width within which adaptive feedback ANC is effective. Fig. 9 shows the attenuation of the FxLMS algorithm and the proposed algorithm on the different frequency ranges. As it is expected the FxLMS algorithm is only effective for broad-band noise with band width of less than 500 Hz. However, the proposed algorithm is still effective when the band width of the noise is about 1250 Hz.

VI. CONCLUSION

In this paper a new structure for adaptive feedback ANC was developed. This structure compensates the secondary path effect in a novel way that improves the convergence rate and band width within which adaptive feedback ANC is efficient. It was shown that the minimum phase component of the secondary path can be directly compensated by filtering the anti-noise signal through inverse of minimum phase component of the secondary path. For compensating the remained component of the secondary path, a new algorithm which we called it All Pass FxLMS algorithm was introduced.

ACKNOWLEDGMENT

The authors would like to thank the University of Auckland for the funding provided for this research.

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