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Localization Using Iterative Angle of Arrival Method Sharing Snapshots of Coherent Subarrays

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Abstract—**In this paper, we propose a localization method using iterative AOA (Angle of Arrival) method sharing snapshots of coherent subarrays. Generally, the conventional AOA method requires many antenna elements to improve the localization accuracy. Thus, AOA method is restricted in some applications because array antenna used in receivers is large. The proposed method improves localization accuracy without increasing elements of antenna arrays, and thus the lower costs and smaller devices are expected. Simulation results demonstrate that the localization accuracy of the proposed method is superior to that of the conventional method using the same number of antennas. Moreover, the proposed method shows the localization accuracy almost identical to that of the conventional method using array antennas with more elements.**

I. Introduction

Localization of sources is attracting a great deal of interest in mobile communications and other many applications. Recently, GPS (Global Positioning System) is used in various applications, such as location information service of cellular phone and car navigation system. However, nodes require to equip with exclusive receivers that are expensive. More importantly, GPS is unavailable indoor or underground. Thus, the localization technique alternative to GPS is attractive [1]–[3].

Accurate indoor localization plays an important role in home safety, public services, and other commercial or military applications [4]. In recent years, indoor localization has drawn increasing interests from academia and industry. In commercial applications, there is an increasing demand of indoor localization systems for tracking persons with special needs, such as elders and children who may be away from visual supervision. Other applications need the solutions to trace mobile devices in sensor networks, or locating accurately in demand portable equipments in hospitals and laboratories. In public safety and military operations, indoor localization systems can be used in navigating and coordinating police officers, firefighters or soldiers to complete their missions inside buildings.

Various alternative localization schemes have been researched and they can be classified to two categories: lateration using distance information by more than two receivers and angulation using direction information by more than one.

TDOA (Time Difference of Arrival) is used to estimate the distance from propagation times through different receivers [5]. RSS (Received Signal Strength) method uses the knowledge of the transmitter power, the path loss model, and the power of the received signal to determine the distance of the receiver from the transmitter [6]. For lateration, a node estimates the distances from three or more beacons to compute its location.

AOA method uses array antenna to estimate direction of arrival and at least two subarrays are required to localize sources [7]. This method is restricted in some applications, because array antenna used in receivers is large. The accuracy of AOA depends on the number of antennas, thus it requires more antennas to improve the accuracy.

In this paper, we propose an iterative localization method based on AOA with compact receivers and low costs. This method requires at least two subarrays each composed of some antennas like the general AOA method. The proposed method can improve the location accuracy without increasing antennas, specifically, sharing the snapshots of each subarray. In other words, it does not increase real elements but increases virtual elements. By localizing iteratively to update directions and distances, the accuracy is improved.

The rest of the paper is organized as follows: section II reviews the conventional localization method using AOA, and section III discusses MUSIC (Multiple Signal Classification) that is the major AOA estimation method. The proposed iterative AOA method is presented in section IV. The performance of the proposed methods compared to some conventional methods is evaluated in section V. We conclude our paper in section VI.

II. General Localization using AOA

AOA method uses array antenna to estimate direction of arrival and more than two subarrays are required to localize sources. Assume that there is a sufficient distance between sources and each subarray, exactly $r \geq 2D^2/\lambda$ [8], where r is

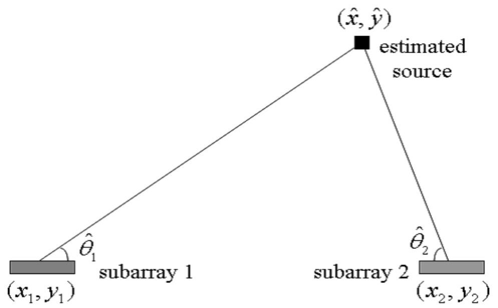

Fig. 1. General AOA method.

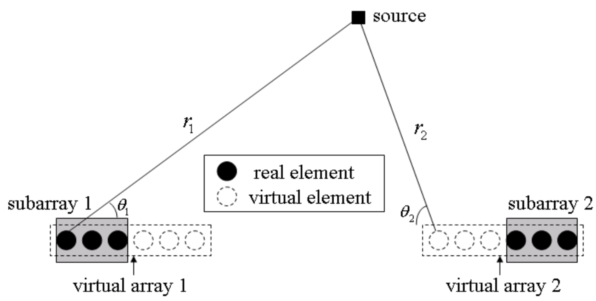

Fig. 2. Proposed AOA method

a distance between source and subarray, D is array aperture, and λ is wavelength.

As shown in Fig. 1, we consider that there are two subarrays and one source in the field. Each subarray estimates signal directions $\hat{\theta}_1$, $\hat{\theta}_2$. Let (x_k, y_k) be the phase center location of subarray k and $(\hat{x}, \hat{y})$ be the estimated location of source, then two lines are respectively written by,

$$\hat{y} - y_1 = (\hat{x} - x_1)\tan\hat{\theta}_1 \quad (1)$$

$$\hat{y} - y_2 = (\hat{x} - x_2)\tan\hat{\theta}_2 \quad (2)$$

From eqs. (1) and (2), $(\hat{x}, \hat{y})$ can be solved as

$$\begin{cases} \hat{x} = \frac{x_1 \tan\hat{\theta}_1 - x_2 \tan\hat{\theta}_2 + y_2 - y_1}{\tan\hat{\theta}_1 - \tan\hat{\theta}_2} \\ \hat{y} = \frac{(x_1 - x_2)\tan\hat{\theta}_1 \tan\hat{\theta}_2 + y_2 \tan\hat{\theta}_1 - y_1 \tan\hat{\theta}_2}{\tan\hat{\theta}_1 - \tan\hat{\theta}_2} \end{cases} \quad (3)$$

Two non-parallel lines are sufficient to locate a position on a plane. How accurate the position is depends on the estimation accuracies of $\hat{\theta}_1$ and $\hat{\theta}_2$. With more than three subarrays, multiple intersection points are available, and one point is selected by some methods [7], for example, *mean aggregation*, etc.

AOA is estimated by MUSIC [9], ESPRIT [10], and so on. In this paper, we choose MUSIC for its simplicity.

III. AOA Estimation by MUSIC

The array snapshots of each subarray composed of M elements at time t radiating L narrowband sources, can be modeled as

$$\mathbf{X}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{S}(t) + \mathbf{N}(t), \quad t = 1, 2, \ldots, N \quad (4)$$

where

$$\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_L)] \quad (5)$$

$$\mathbf{S}(t) = [s_1(t), \ldots, s_L(t)]^T \quad (6)$$

$$\mathbf{N}(t) = [n_1(t), \ldots, n_M(t)]^T \quad (7)$$

and $[\cdot]^T$ is transpose operator. $\mathbf{a}(\theta_l) = [a_1(\theta_l), \ldots, a_M(\theta_l)]$ is steering vector, meaning phase difference between elements for signal direction from θ_l $(l = 1, \ldots, L)$. When the subarray geometry is ULA (uniform linear array) whose interelement spacing is d, the mth array response is

$$a_m(\theta) = e^{j(2\pi/\lambda)(m-1)d\sin\theta}, \quad m = 1, 2, \ldots, M \quad (8)$$

$s_l(t)$ is the lth signal waveform, $n_m(t)$ is the addtional white noise of the mth element, and N is the number of snapshots.

From the snapshots (4), the sample estimate of the correlation matrix is given by

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N} \mathbf{X}(t)\mathbf{X}^H(t) \quad (9)$$

where $[\cdot]^H$ is conjugate transpose operator.

Write the eigendecomposition of $\hat{\mathbf{R}}$ as

$$\hat{\mathbf{R}} = \hat{\mathbf{E}}_S\hat{\mathbf{\Lambda}}_S\hat{\mathbf{E}}_S^H + \hat{\mathbf{E}}_N\hat{\mathbf{\Lambda}}_N\hat{\mathbf{E}}_N^H \quad (10)$$

where $\hat{\mathbf{\Lambda}}_S$ is a $L \times L$ diagonal matrix containing the largest eigenvalues of $\hat{\mathbf{R}}$ and $\hat{\mathbf{E}}_S$, called signal subspace, is an $M \times L$ matrix that contains the eigenvectors corresponding to the L largest eigenvalues. $\hat{\mathbf{\Lambda}}_N$ is a $M \times (M - L)$ diagonal matrix containing the smallest eigenvalues of $\hat{\mathbf{R}}$ and $\hat{\mathbf{E}}_N$, called noise subspace, is an $M \times (M - L)$ matrix that contains the eigenvectors corresponding to the $M - L$ smallest eigenvalues.

MUSIC algorithm [9] is based on the property that any vector lying on the signal subspace $\mathbf{E}_S$ is orthogonal to the noise subspace $\mathbf{E}_N$. Then, we will have

$$\mathbf{a}^H(\theta_l)\mathbf{E}_N = \mathbf{0} \quad (11)$$

for any $l = 1, \ldots, L$.

Once the noise subspace is estimated by eqs.(10), AOA can be obtained by searching for peaks in the MUSIC spectrum given by

$$P_{\mathrm{MUSIC}}(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\hat{\mathbf{E}}_N\hat{\mathbf{E}}_N^H\mathbf{a}(\theta)} \quad (12)$$

IV. Proposed Method

We propose an iterative localization method in this paper. In the proposed method, the source location is estimated roughly in initial estimation and the location is updated in update estimation. Let us consider that there are two ULA subarrays and virtual arrays in the field as Fig. 2. Each virtual array is composed of self-subarray elements and other subarray elements.

The array snapshots of each subarray composed of M elements at time t can be modeled as

$$\mathbf{X}_1(t) = \mathbf{A}_1(\theta)\mathbf{S}(t) + \mathbf{N}_1(t) \quad (13)$$

$$\mathbf{X}_2(t) = \mathbf{A}_2(\theta)\mathbf{S}(t) + \mathbf{N}_2(t) \quad (14)$$

where $\mathbf{X}_k(t), \mathbf{A}_k(\theta), \mathbf{N}_k(t)$ are the snapshots, steering matrix, additional white noise of subarray k, respectively.

When the reference point is source point, the mth array response (8) in virtual array k can be written as

$$v_m^k(\theta) = a_m(\theta)b(r_k) \quad (15)$$

where

$$a_m(\theta) = e^{j(2\pi/\lambda)(m-1)d\sin\theta} \quad (16)$$

$$b(r_k) = e^{j(2\pi/\lambda)r_k} \quad (17)$$

Note that $a_m(\theta)$ corresponds to array response in virtual ULA k and $b(r_k)$ corresponds to phase shift from source to virtual array k.

In the proposed method, we assume that symbol synchronization between source and subarrays is ideal, and that the snapshots of first element in virtual array 1 equals to those of first element in virtual array 2. In other words, the snapshots of the reference point in each virtual array are same. Thus, eq. (15) is simply rewritten as

$$v_m^k(\theta) = e^{j(2\pi/\lambda)(m-1)d\sin\theta} \quad (18)$$

A. Initial estimation

First, each subarray uses own correlation matrix to estimate AOA given by

$$\hat{\mathbf{R}}_1 = \frac{1}{N}\sum_{t=1}^{N}\mathbf{X}_1(t)\mathbf{X}_1^H(t) \quad (19)$$

$$\hat{\mathbf{R}}_2 = \frac{1}{N}\sum_{t=1}^{N}\mathbf{X}_2(t)\mathbf{X}_2^H(t) \quad (20)$$

Directions $\hat{\theta}_1^{(1)}, \hat{\theta}_2^{(1)}$ are obtained by (10), (12) and source location $(\hat{x}, \hat{y})$ are obtained by (3). This is the same algorithm as the general AOA method.

B. Update estimation

Next, distances between source and each subarray $\hat{r}_1, \hat{r}_2$ are computed from estimated source location and known each subarray location. We have, now, rough direction and distance about source. Then, sharing the array snapshots

$$\mathbf{X}_v(t) = \begin{bmatrix} \mathbf{X}_1(t) \\ \mathbf{X}_2(t) \end{bmatrix} \quad (21)$$

This means that the dimension of each subarray snapshots increases from $M \times 1$ to $2M \times 1$. Each subarray uses extended correlation matrix to estimate AOA given by

$$\hat{\mathbf{R}}_v = \frac{1}{N}\sum_{t=1}^{N}\mathbf{X}_v(t)\mathbf{X}_v^H(t) \quad (22)$$

and the array response of virtual array 1 is given by

$$v_m^1(\theta) = \begin{cases} (1/\hat{r}_1)e^{j(2\pi/\lambda)(m-1)d\sin\theta}, & \text{in subarray 1} \\ (1/\hat{r}_2)e^{j(2\pi/\lambda)(m-1)d\sin\hat{\theta}_2^{(1)}}, & \text{in subarray 2} \end{cases} \quad (23)$$

where $(1/\hat{r}_k)$ is inverse of the distance between source and subarray k that means signal fading coefficient. Note that θ is the variable and $\hat{\theta}_2$ is the constant that is updated. Assume that $\hat{\mathbf{U}}_N$ is the noise subspace of $\hat{\mathbf{R}}_v$ and $\mathbf{v}^1(\theta) = [v_1^1(\theta), v_2^1(\theta), \ldots, v_{2M}^1(\theta)]$ is the steering vector, MUSIC spectrum (12) in virtual array 1 is given by

$$P_{\mathrm{MUSIC}}^1(\theta) = \frac{\mathbf{v}^{1H}(\theta)\mathbf{v}^1(\theta)}{\mathbf{v}^{1H}(\theta)\hat{\mathbf{U}}_N\hat{\mathbf{U}}_N^H\mathbf{v}^1(\theta)} \quad (24)$$

Similary, MUSIC spectrum in virtual array 2 is given by

$$P_{\mathrm{MUSIC}}^2(\theta) = \frac{\mathbf{v}^{2H}(\theta)\mathbf{v}^2(\theta)}{\mathbf{v}^{2H}(\theta)\hat{\mathbf{U}}_N\hat{\mathbf{U}}_N^H\mathbf{v}^2(\theta)} \quad (25)$$

Note that signal subspace of each virtual array $\hat{\mathbf{U}}_N$ is same because of using the same snapshots $\mathbf{X}_v(t)$. From eqs. (24) and (25) we get new directions $\hat{\theta}_1^{(2)}$ and $\hat{\theta}_2^{(2)}$, thus the new source location is estimated. This is second update. Then, the proposed method iteratively updates the directions and estimates the source locations.

V. Simulation Results

In this section, we examine the performance of our proposed method. In all numerical examples, the location of a source whose frequency is single and each subarray is as Fig. 3. The source is located at random from 16 points. and the geometry of each subarray is ULA whose interelement spacing is $d = \lambda/2$. AOA of each subarray is estimated by MUSIC and the number of snapshots is 100. All results are averaged over 10000 simulation runs.

We compare the proposed method to three conventional methods for the number of antennas. Conv. 1 (4×2) uses two subarrays each composed of four elements, Conv. 2 (5×2) uses two subarrays each composed of five elements, and Conv. 3 (4×4) uses four subarrays each composed of four elements. For Conv. 3, the source location is determined by *mean aggregation* for 4 intersection points [7]. The goal of our proposed method is to improve location accuracy without increasing the number of antennas. Thus, Prop. (4×2) uses two subarrays each composed of four elements, which is the same

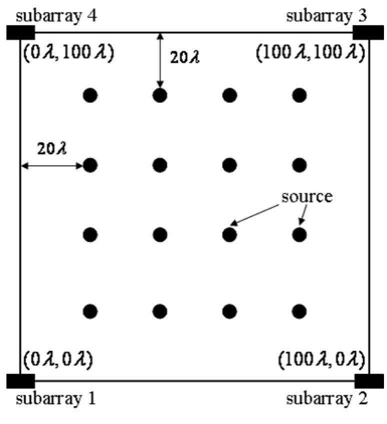

Fig. 3. The location of a source (random) and each subarray.

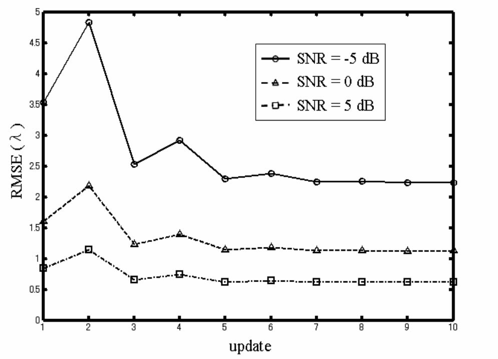

Fig. 4. Location RMSEs versus update times of proposal.

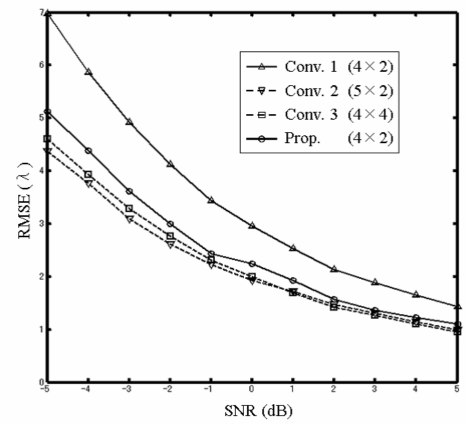

Fig. 5. Location RMSEs versus SNR.

number of antennas, subarrays and elements of subarrays, as Conv. 1.

In Fig. 4, the RMSEs (root mean square errors) versus the update times are shown in each SNR for Prop.. Update times = 1 means initial estimation explained in Section IV-A. From Fig. 4, the RMSEs for Prop. are improved as the update time increases, and after 6 times they do not change. We decided maximum update times is 10 in this simulation.

In Fig. 5, the RMSEs of the location estimates for all the methods versus SNR (signal to noise ratio) are shown. Prop. performs asymptotically close to Conv. 2 and Conv. 3, and outperforms Conv. 1. This is because Prop. uses more number of snapshots than Conv. 1. Prop. shows the more robustness, particularly in low SNR. We stress that in contrast to Prop., Conv. 2 and Conv. 3 use more antennas.

In Fig. 6, the PDFs (probability density function) of location RMSEs versus the error distance, 10λ intervals, are shown. The PDF of Prop. in the small errors, less than 10λ, is larger than that of Conv. 2, whereas in the large errors, more than 100λ, is also larger. This is because in Prop., AOA is estimated by using the parameters (directions and distances) estimated in the previous update. Thus, if the errors of previous parameters are large, Prop. uses the erroneous parameters updated for the worse.

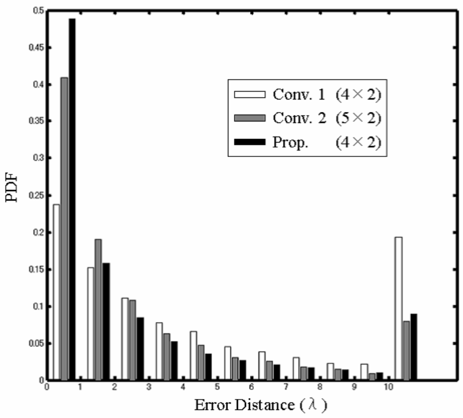

Fig. 6. PDFs of Location RMSEs versus the error distance.

VI. Conclusion

In this paper, we propose a localization method using iterative AOA method sharing snapshots of coherent subarrays that does not increase antenna elements. The proposed method performs asymptotically close to the conventional one using more antennas, and outperforms the conventional one using the same number of antennas.

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