An improved adaptive wiener filtering algorithm for super-resolution

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ABSTRACT

This paper presents an improved adaptive Wiener filtering algorithm for super-resolution reconstruction. When interpolating the high resolution pixels, the proposed algorithm locally adjusts the correlation model of the pixels by taking edge information into account. Simulation results show that the proposed algorithm produces SR outputs of better quality, both subjectively and objectively, as compared with the conventional approach.

***Index Terms* — Adaptive** Wiener Filtering, Super-Resolution, Non-Uniform interpolation.

I. INTRODUCTION

Super-Resolution (SR) is an image fusion issue in which multiple low resolution images are fused into a higher resolution image. Due to its ill-pose nature, there is no unique solution to solve such a fusion problem. Many algorithms [1-8] have been proposed over last two decades. The problem was initially addressed in frequency-domain by Huang [1], and extended later by Bose [2]. Although the frequency-domain approach is computationally inexpensive and intuitively simple, it can only handle translation motion between consecutive images and hence limits its applications. Another approach to solve the SR problem is to handle it as an interpolation and restoration problem in the spatial domain [3-8].

Recently, how to realize high quality SR reconstruction with less computation has attracted many efforts. Hardie [3] proposed a fast SR algorithm by using an adaptive wiener filter. Observed pixels from successive low-resolution (LR) frames (referred to as LR pixels hereafter) are initially registered onto a common high-resolution (HR) grid by using sub-pixel registration. In contrast to some previous techniques, the registered positions of the LR pixels in the HR grid are not quantized to the nearest finite grid in Hardie's algorithm (AWF). The pixels in the HR frame to be reconstructed (referred to as HR pixels hereafter) are estimated as a weighted sum of the registered LR pixels in the same local region. Based on a simple statistical model, the weights of the LR pixels are designed to minimize the mean squared error of the estimation. Consequently, the weight for each involved registered LR pixel is a function of (1) its distance from the HR pixel to be estimated and (2) the local sample variance in the local region.

In this paper, we extend the idea of AWF to improve its performance in terms of output quality. Besides making the Wiener filter adaptive to the local variance of LR pixels, the proposed algorithm adapts the filter to the nature of the local region. The remainder of this paper is organized as follows. A brief description of AWF is first presented in Section II. Section III highlights the proposed modification to AWF. Section IV presents some simulation results. Finally, a brief concluding remark is provided in Section V.

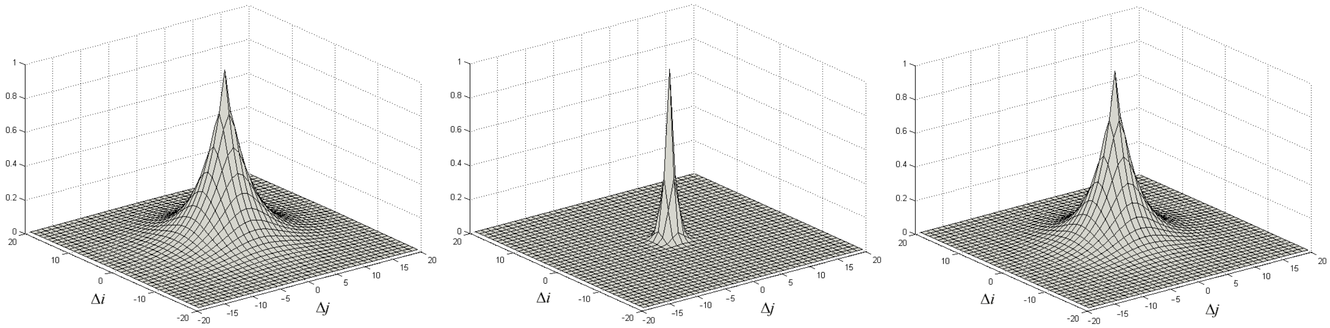
II. CONVENTIONAL AWF

Super resolution aims to enhance the spatial resolution of an image with the fusion of multiple LR images. Observed LR pixels in each LR image are noisy samples of a blurred translated version of the original HR image.

To start the process, all LR pixels in successive LR images are registered onto a higher resolution grid. Through a spatial sliding window covering a $3d\times3d$ region of the high resolution grid, where d is the ratio of the target resolution of the reconstructed HR image to the original resolution of a LR image, AWF collects all LR pixels registered in the window to estimate the $d\times d$ HR pixels in the center of the window. Without losing the generality, we assume that the number of registered LR pixels in the window is L and the total number of HR pixels to be estimated with these L LR pixels is N. In AWF, the N HR pixels, denoted as $\vec{\mathrm{x}}=[x_1,x_2,\ldots x_N]^T$ in vector form, are estimated as

$$\hat{x}_m=\frac{\sum_{k=1}^{L}\omega_{mk}y_k}{\sum_{k=1}^{L}\omega_{mk}} \qquad \text{for } m=1,2\ldots N \quad (1)$$

where $\vec{\mathrm{y}}=[y_1,y_2,\ldots y_L]^T$ are the registered LR pixels, $\hat{x}_m$ is the estimate of the m^{th} HR pixel and $\omega_{mk}/\sum_{n=1}^{L}\omega_{mn}$ is the normalized weight of the k^{th} LR pixel when estimating $\hat{x}_m$. Weights ω_{mk} are determined with



(a) AWF (b) Ours - edge region (c) Ours - smooth region

Fig.1 Correlation models of HR pixels

$$W = \begin{bmatrix} \omega_{11} & \omega_{21} & \cdots & \omega_{N1} \\ \omega_{12} & \omega_{22} & & \omega_{N2} \\ \vdots & & \ddots & \vdots \\ \omega_{1L} & \omega_{2L} & \cdots & \omega_{NL} \end{bmatrix} = R^{-1} \cdot P \tag{2}$$

where R is the auto-correlation matrix of $\{ y_k \mid k=1,\ldots L\}$ and P is the cross-correlation matrix of $\{ y_k \mid k=1,\ldots L\}$ and $\{ x_m \mid m=1,\ldots N\}$.

By considering that the observed LR pixel y_k is corrupted with an additive zero mean random noise n and its noise-free version is $f_k = y_k - n$, we have

$$R = E\{\bar{y} \cdot \bar{y}^T\} = E\{\bar{f} \cdot \bar{f}^T\} + \sigma_n^2 \cdot \mathrm{I} \tag{3}$$

and $$P = E\{\bar{y} \cdot \bar{x}^T\} = E\{\bar{f} \cdot \bar{x}^T\} \tag{4}$$

where $E\{\bullet\}$ is the expectation operator and σ_n^2 is the local variance of the noise.

In theory, f_k are samples of $\{f(i,j)\}$, a blurred version of the original HR image $\{x(i,j)\}$. In formulation, we have $f(i,j) = x(i,j) * b(i.j)$, where b is the blurring function and (i,j) denotes a position in the HR grid. Accordingly, the cross-correlation of $x(i,j)$ and $f(i,j)$ and the auto-correlation of $f(i,j)$ can be, respectively, determined by

$$r_{fx}(i,j) = r_{xx}(i,j) * b(i,j) \tag{5}$$

and $$r_{ff}(i,j) = r_{xx}(i,j) * b(i,j) * b(-i,-j) \tag{6}$$

It is assumed in AWF that the correlation between two HR pixels can be modeled as

$$r_{xx}(\Delta i, \Delta j) = \sigma_x{}^2 \cdot \rho^{\sqrt{\Delta i^2 + \Delta j^2}} \tag{7}$$

where (Δi , Δj) is the distance of the two HR pixels, ρ (=0.75) is a tuning parameter that controls the decay of the autocorrelation with distance and σ_x^2 is the local variance of the HR image. In particular, σ_x^2 is estimated as

$$\sigma_x^2 = \max\left\{(\sigma_y^2 - \sigma_n^2) / C, 0\right\} \tag{8}$$

where σ_y^2 is the variance of $\{ y_k \mid k=1,\ldots L\}$ and

$$C = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho^{\sqrt{i^2 + j^2}} \cdot (b(i,j) * b(-i,-j)) \cdot di \cdot dj \tag{9}$$

Elements of $E\{\bar{f} \cdot \bar{f}^T\}$ and $E\{\bar{f} \cdot \bar{x}^T\}$ in eqns. (3) and (4) and hence W can then be determined with $r_{ff}(\Delta i, \Delta j)$ and $r_{fx}(\Delta i, \Delta j)$, where (Δi , Δj) is the distance of the two involved pixels in the HR grid.

III. PROPOSED ALGORITHM

In AWF, a HR pixel is estimated to be a weighted sum of the registered LR pixels in its local region. From Section II, one can see that the weight for each involved registered LR pixel is a function of (1) its distance from the HR pixel to be estimated and (2) the local sample variance in the local region. The model of $r_{xx}(\Delta i, \Delta j)$ specified in eqn. (7) plays a significant role to determine the value of the weight.

AWF's correlation model $r_{xx}(\Delta i, \Delta j)$ adapts to the local sample variance only and does not take the edge characteristics into account. Obviously, pixels are more correlated in a non-edge region than an edge region. An edge breaks the correlation between the pixels in different sides of the edge.

By considering the aforementioned factor, two separate correlation models for edge and smooth regions are used in the proposed algorithm. The general form of the two models can be defined as

$$r_{xx}(\Delta i, \Delta j) = \sigma_x^2 \exp(-\tfrac{1}{\kappa}(\Delta i^2 + \Delta j^2)^q) \tag{10}$$

where q and κ are parameters to model different levels of correlation among samples and they can be determined via an empirical study. In our study, the two parameters are selected to be (q,κ) = (0.5,3) for smooth regions and (q, κ) = (0.8,3) for edge regions. As our correlation model is different from AWF's, when estimating σ_x^2 with eqn. (8), the parameter C should be modified as

$$C = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{\kappa}(i^2+j^2)^q} \cdot (b(i,j) * b(-i,-j)) \cdot di \cdot dj \qquad (11)$$

Fig. 1 shows the autocorrelation models used in AWF and our proposed algorithm for comparison.

To select an appropriate correlation model, a region should be classified with an edge detection scheme. In the proposed algorithm, a Sobel operator is applied to the reference LR image the grid of which aligns with the HR grid to define a binary edge map for the reference LR image. Accordingly, when estimating the HR pixels of a particular region, each of the involved registered LR pixels coming from the reference image is either classified to be an edge or non-edge pixel. If all of them are non-edge pixels, the region will be classified to be a smooth region or else an edge region.

IV. SIMULATION RESULTS

Simulations were carried out to evaluate the improvement of the proposed algorithm with respect to AWF. A set of 8-bit gray level images shown in Fig. 2 were used as original HR images in the simulation. For each one of them, a sequence of 10 LR images were generated as follows. First of all, a sequence of real value displacement vectors (Δ_x, Δ_y) the values of which were bounded by ±5 were randomly generated. The HR image was then translationally shifted by subpixel spacings using bicubic interpolation, blurred with a Gaussian filter of size 5×5, down-sampled by 5 and 5 in the horizontal and vertical directions, respectively, and finally corrupted with additive zero-mean random noise of variance σ_n^2. The translational shift for the k^{th} image in the sequence was set to be the accumulated sum of the first k-1 aforementioned displacement vectors.

In the simulation, the original HR images were estimated with their corresponding LR sequences. The initial sub-pixel registration step was carried out with the algorithm proposed in [9].

Table 1 shows the performance achieved with different SR algorithms in terms of *PSNR* under different noise condition. In particular, *PSNR* is defined as

$$PSNR = 10 \cdot \log_{10} \frac{255^2}{\frac{1}{N_0}\sum_{(i,j)}(x(i,j) - \hat{x}(i,j))^2} \text{ in dB} \quad (12)$$

where x and $\hat{x}$ are, respectively, the original HR image and its estimate, and N_0 is the total number of image pixels involved in the comparison.

Figs. 3 and 4 show parts of the simulation results of different SR algorithms for visual comparison. One can see that the proposed algorithm provides sharper interpolation results than AWF.

Fig.2 Set of testing images (referred to as *Pentagon, Beacon, Boat, Statue, Wall, Motor* and *Flower*, from top-to-bottom and left-to-right). Image size : *Pentagon* - 2386×2404 , Others - 512×768 or 768×512

	$\sigma_n^2 = 25$			$\sigma_n^2 = 50$		
methods	Bicubic	AWF[3]	Ours	Bicubic	AWF[3]	Ours
Pentagon	17.27	22.48	23.42	17.21	22.27	23.07
Wall	19.08	24.42	24.99	19.02	24.14	24.56
Motor	18.31	23.82	24.72	18.21	23.59	24.30
Flower	21.14	28.93	29.64	21.00	28.51	28.79
Boat	22.17	28.49	29.00	21.99	28.14	28.37
Beacon	18.06	23.53	24.48	17.97	23.33	24.13
Statue	22.21	28.12	28.57	22.00	27.84	27.72
Average	19.75	25.69	26.40	19.63	25.40	25.85
Variance	4.23	7.38	6.56	4.04	7.02	5.54

Table 1 *PSNR* performance of various SR algorithms including Bicubic, AWF(Q=∞)[3] and the proposed.

V. CONCLUSIONS

A modification to a recently proposed SR algorithm [3] is presented in this paper. The proposed modification classifies image regions and accordingly adapts the correlation model of HR pixels to reflect the real situation when determining the weights of the involved LR pixels to estimate a HR pixel. As a consequence, the improved algorithm adapts the estimation of HR pixels to both the local sample variance and the local edge characteristics. Simulation results show that the modification improves the SR performance both objectively and subjectively.

VI. ACKNOWLEDGEMENT

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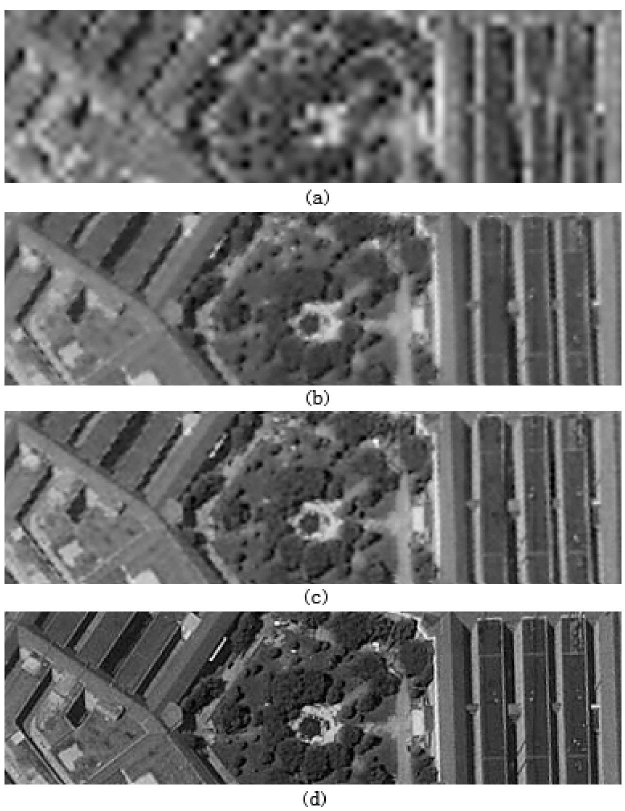

Fig.3 Parts of the SR results of 'Pentagon': (a) Bicubic, (b) AWF[3], (c) the proposed and (d) the original

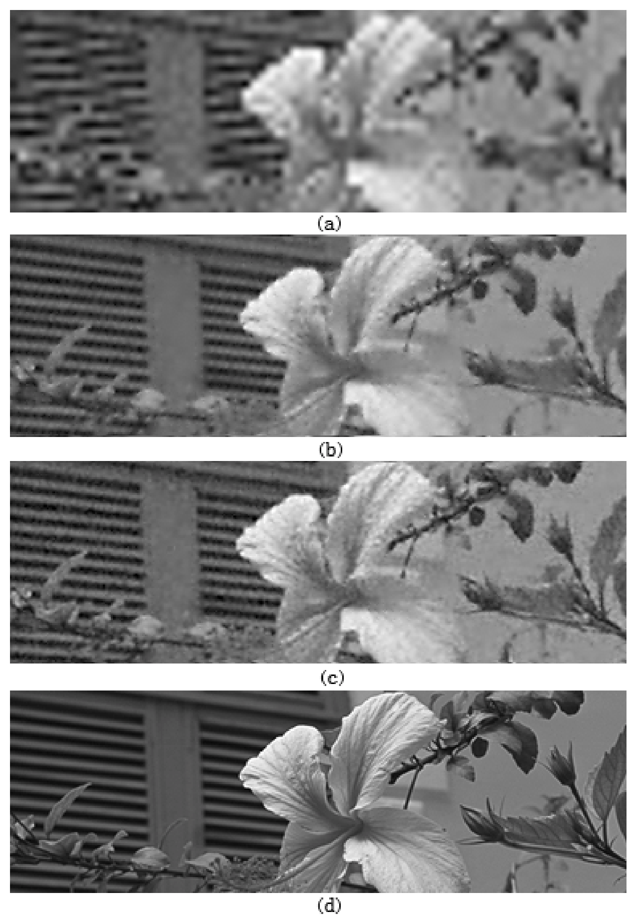

Fig.4 Parts of the SR results of 'Kodak07': (a)Bicubic, (b)AWF[3], (c) the proposed and (d) the original