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Quality Estimation of Fractal Coded Images by Using Level of Self-Similarity

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Abstract—**Fractal image coding is a technique for coding digital images. It can provide the compressed images with higher quality than JPEG compression at ultra low bit-rates. However, we cannot use it as a practical coding technique. One of the reasons is that the quality of some of the compressed images is not sufficient for practical applications. Moreover, what is more inconvenient is that we cannot know the compressed image quality of a given image unless we actually encode and decode it which takes a lot of time. Therefore, we decided to try to resolve this problem by implementing the following steps: (i) we find the important image features which relate to the quality of the compressed images, and we establish an estimation method of the compressed image quality by using the important image feature values. (ii) Then, based on the estimated quality by this method, we apply the fractal image coding to each image adaptively. This paper presents the above step (i).**

I. Introduction

Fractal image coding[1], [2], [3] based on an iterated function system (IFS) is one of the coding techniques for digital images. It can achieve high image compression by utilizing the self-similarity of the images. The quality of the compressed images is higher than the quality of JPEG images at ultra low bit-rates. In addition to the high compression performance, it has the advantages of enabling images to be decoded in a few seconds and at arbitrary resolution.

However, the fractal image coding has a disadvantage that the quality of some compressed images is not sufficient for practical applications. Additionally, what is more inconvenient is that we cannot know whether the compressed image quality for a given image is low unless we actually encode and decode it which takes a lot of time. This problem must be resolved in order for fractal image coding to become a practical technique. We therefore decided to try to resolve this problem by the following steps.

(i) We investigate the relationships between various features of the original image and quality of the compressed image, and find the important features which relate to the compressed image quality. Then by using these important image features, we establish an estimation method of the compressed image quality.
(ii) By using the estimation method, we can predict the quality of the compressed images without actually encoding images, and discriminate suitable images from unsuitable images for fractal image coding. Then based on the suitability, we apply the fractal image coding to each image adaptively.

In this paper, the above (i) is presented.

II. Fractal image coding

We introduce basic procedures for the fractal image coding[1], [2], [3]. Each procedure is executed as follows:

(i) A given image is partitioned into non-overlapping blocks (range blocks, hereafter) of size $B \times B$ and into arbitrarily located blocks (domain blocks, hereafter) of size $2B \times 2B$. The range blocks are numbered from 1 to M, and denoted by $R_i (i = 1, 2, \cdots, M)$. Similarly, the domain blocks are numbered from 1 to N, and denoted by $D_j (j = 1, 2, \cdots, N)$.
(ii) Each domain block D_j is contracted by the affine transformation τ_{ij} defined as follows:

$$\begin{aligned} &\begin{pmatrix} \tilde{p} \\ \tilde{q} \\ \tilde{v}(\tilde{p}, \tilde{q}) \end{pmatrix} \\ &= \tau_{ij} \begin{pmatrix} p \\ q \\ v(p,q) \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2}\cos\theta & -\frac{1}{2}\sin\theta \times \lambda & 0 \\ \frac{1}{2}\sin\theta & \frac{1}{2}\cos\theta \times \lambda & 0 \\ 0 & 0 & a \end{pmatrix} \begin{pmatrix} p \\ q \\ v(p,q) \end{pmatrix} \\ &\quad + \begin{pmatrix} g \\ h \\ b \end{pmatrix} \end{aligned} \tag{1}$$

where $\theta \in \{0°, 90°, 180°, 270°\}$ and $\lambda \in \{1, -1\}$. (p, q) and $(\tilde{p}, \tilde{q})$ are the coordinates of the pixels in the domain block D_j and the obtained block $\tau_{ij}(D_j)$, respectively. Also, $v(p, q)$ and $\tilde{v}(\tilde{p}, \tilde{q})$ are their intensity values. The parameters $g, h \in \{0, B - 1\}$ are offsets for the coordinates. The parameters a and b are called a scaling coefficient and an offset, respectively.

(iii) For each range block R_i, we find the best contracted domain block D_i^{opt} among the contracted domain blocks obtained by using various θ, λ, a and b in (1). The best contracted domain block D_i^{opt} satisfies the following equation:

$$d(R_i, D_i^{\mathrm{opt}}) = \min_{\theta,\lambda,a,b,j} d(R_i, \tau_{ij}(D_j)). \qquad (2)$$

In the above equation, the distortion measure $d(X, Y)$ is the mean square error (MSE) between X and Y.

(iv) For each range block, the codes which provide the best contracted domain block are encoded. The codes are called the IFS codes. Concretely the IFS codes are composed of the parameters (θ_i^{opt}, λ_i^{opt}, a_i^{opt}, b_i^{opt}) of the best affine transformation and the parameters (x_i^{opt}, y_i^{opt}) to indicate the location of the best matched domain block.

III. Proposed Criterion for Suitability of the Fractal Image Coding

We have investigated the relationships between the quality of the compressed images and features of the original images: fractal dimension, self-similarity and the correlation coefficient between neighboring pixels. In the following sections, the calculation method of the fractal dimension and the calculation method of the level of self-similarity are described.

A. Calculation of level of fractal dimension

Fractal dimension is a number, generally a non-integer number, that represents the self-similarity or complexity of a figure. For example, the fractal dimension of a linear figure is between 1.0 and 2.0, and the fractal dimension of an image with shading is between 2.0 and 3.0.

Various methods for estimating fractal dimension have been proposed [4], [5], [6], [7]. In this study, we used the box-counting method[4]. Therefore, we explain basic procedures for the box-counting method by using Fig. 1. Each procedure is executed as follows:

(i) In a given image, the coordinates of the pixels are denoted by (x, y) and their intensity values are $I(x, y)$.

(ii) The object of $(x, y, I(x, y))$ is covered by the cubes of size $r \times r \times r (r = 1, \cdots, r_{\max})$ and the number of the required cubes are denoted by the $N(r)$.

(iii) $N(r)$, r and the fractal dimension D satisfy the following equation:

$$N(r) \propto r^{-D}. \qquad (3)$$

Therefore, the fractal dimension can be calculated from the slope of the plot of the $\log r$ versus $\log N(r)$.

Because the computation cost in step (ii) increases as the size of cube grows, we stopped at $r = 41$.

B. Calculation of level of self-similarity

Self-similarity of an image means that if one part of an image is enlarged or reduced in size, it has the same shape or complexity as that of a larger part or the whole image. An object that shows self-similarity is called a fractal. One of the simplest examples of the fractal is the Mandelbrot set shown in Fig. 2. Natural images rarely have exact self-similarity, but many have approximate self-similarity.

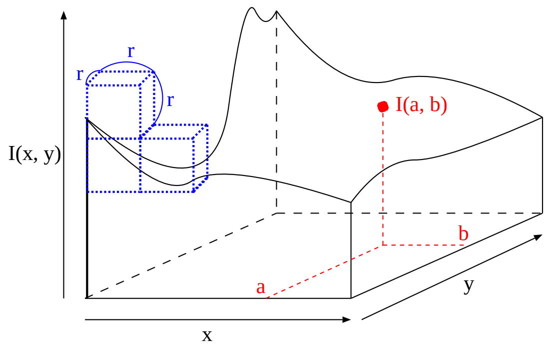

Fig. 1. How to calculate fractal dimension of image by the box-counting method.

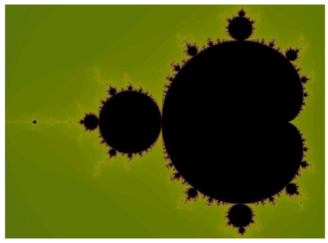

Fig. 2. Mandelbrot set.

Since there are few images that show perfect self-similarity, all points in a log-log graph ($\log r, \log N(r)$) used for estimating fractal dimension rarely lie on a straight line. Hence, fractal dimension is generally estimated from the slope of the regression line. However, if the image has perfect self-similarity, all points in the log-log graph will lie on a straight line. The level of self-similarity can thus be determined by the distance between the data ($\log r, \log N(r)$) and the predicted data by a linear regression model, that is, the degree of fitness to a linear regression model [8]. Therefore, in this paper, the level of self-similarity is obtained as the correlation coefficient between the data ($\log r, \log N(r)$) and the predicted data.

C. Relationships between compressed image quality and image features

An experiment was carried out to verify the relationships between the image features and the compressed image quality obtained by the fractal image coding. The experiment was performed using eight images of 256×256 pixels in size and 8bit gray-level shown in Fig. 3.

We calculated the fractal dimension, the level of self-similarity using the index explained in Sect. III-B and the correlation coefficient between neighboring pixels of the test images. Then, in order to reveal the relationships between image features and compressed image quality, the scatter plots of the image feature value versus the MSE of the compressed

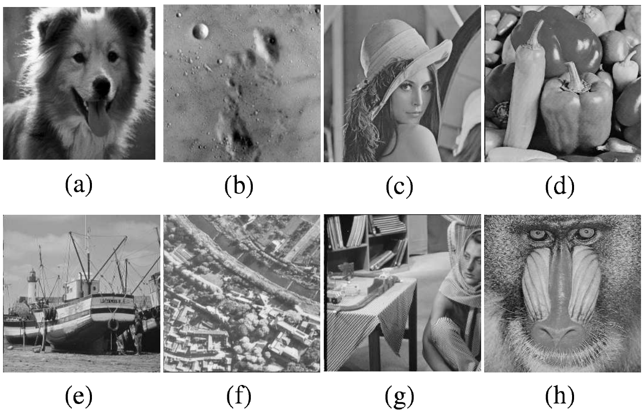

Fig. 3. Target images ((a)collie, (b)moon, (c)lena, (d)peppers, (e)boat, (f)aerial, (g)barbara, (h)mandrill).

images obtained by fractal image coding are shown in Fig. 4. Here, we use the fractal image compression introduced in Sect. II, and the range block size is 8×8 pixels.

As can be seen in Fig. 4(b), the level of self-similarity has a linear relationship with the quality of compressed images obtained by fractal image coding. And from Fig.4(a) and (c), the roughly linear relationships are also observed between other image features and the compressed image quality. However, these results indicate that it is difficult to estimate the compressed image quality from only the level of self-similarity, and it is not possible to determine which images can and cannot be effectively compressed by fractal image coding. We therefore propose a method for accurate estimation of compressed image quality using the fractal dimension and the correlation coefficients of the image in addition to self-similarity.

IV. Estimation of Compressed Image Quality

In the following, a new method for estimating the quality of a compressed image obtained by fractal image coding is described.

A. Estimation method

We propose the following equation that includes the fractal dimension and the correlation coefficient between neighboring pixels as features of the image in addition to self-similarity for estimating the quality of the compressed image:

$$f = \alpha d + \beta s + \gamma c + \delta, \qquad (4)$$

where d is the fractal dimension, s is the level of self-similarity calculated as described in Sect. III-B, c is the correlation coefficient between neighboring pixels, and α, β, γ and δ are constants.

B. Experimental results

Experiments were conducted to confirm the estimation accuracy of the proposed method. The values of the coefficients α, β, γ and δ in (4) were determined by multiple linear regression analysis. The eight images shown in Fig. 3 were used as test

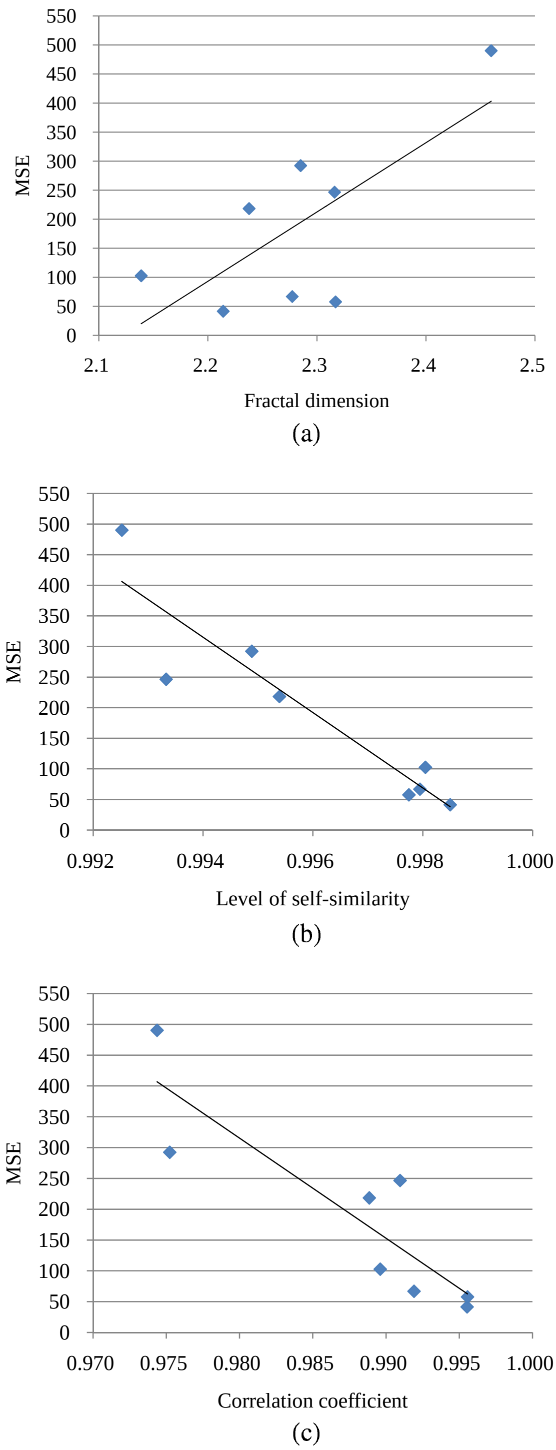

Fig. 4. Scatter plots of the image feature value versus the MSE of compressed images ((a) Fractal dimension, (b) Level of self-similarity, (c) Correlation coefficient).

images. Fractal image coding was applied to each of the test images, and MSE of each of the compressed images was calculated. The MSEs of the compressed images are shown as (i) in Fig. 5. And the estimated MSEs by the proposed method are shown as (ii) in Fig. 5. This figure indicates that the quality

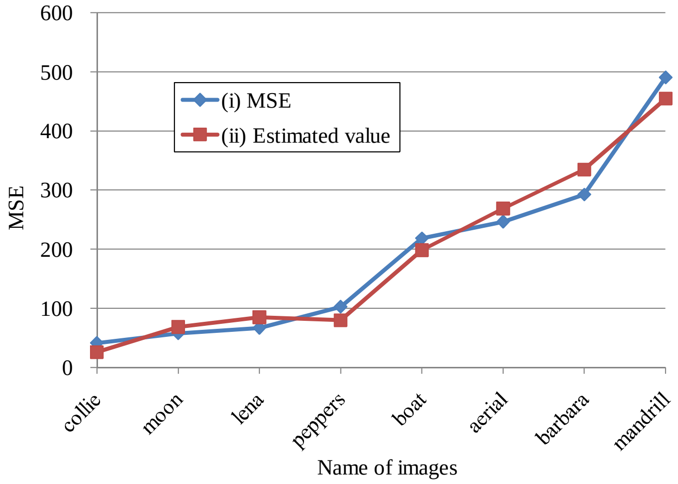

Fig. 5. MSEs of the compressed images and the estimated MSEs by the proposed method

of the compressed image can be estimated with a high level of accuracy by the proposed method.

V. Summary

Fractal image coding enables high compression of image data by utilizing the self-similarity of an image. However, the quality of some compressed images is not sufficient for practical applications, and this problem must be resolved in order for fractal image coding to become a practical technique. As a step towards resolving this problem, we investigated the relationships between the image features and the quality of the compressed image to clarify which images are suitable and which are not suitable for fractal image coding.

An experiment was carried out to verify the relationships between the image features and the compressed image quality by the fractal image coding. And the result showed that there is a close relationship between self-similarity of an image and quality of the compressed image. However, since it was difficult to estimate the quality of the compressed image only from self-similarity of the original image, we proposed a method for estimating quality of the compressed image using additional features of the original image.

Results obtained by applying the proposed method to natural images showed that the quality of the compressed image can be estimated with a high level of accuracy by using this method, confirming the usefulness of this new method.

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