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Analysis of Adaptive Filters in Feedback Cancellation for Sinusoidal Signals

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***Abstract*—Analysis of coupling effects in adaptive filters is presented for feedback cancellation systems in hearing aids. As the incoming signal, a sum of sinusoids in white noise is assumed in this paper. In the proposed scheme two LMS-type adaptive filters are used. The one is set for identifying the impulse response of the feedback path. The other is associated with frequency estimation. We propose a scheme using the constrained LMS algorithm for the adaptive filter for frequency estimation. The stationary point of the proposed algorithm is shown to be the desired ones. We also show that the simultaneous stability near the stationary point is guaranteed. Simulation results show the validity of the theoretical findings.**

I. Introduction

In hearing aids an adaptive filter is used to model the feedback path from the receiver (speaker) to the microphone and cancels its effect. The conventional LMS algorithm is biased, since the input signal and the incoming signal which acts as observation noise in the adaptive filter are correlated due to the feedback path.

In [1], the PEM-AFC (Prediction Error Method-based Adaptive Feedback Canceller) algorithm has been proposed for continuous unbiased cancellation for the case where the incoming signal is an AR (autoregressive) process. But its convergence mechanism to the desired point was not discussed.

In [2], under an assumption that the incoming signal is a sum of sinusoids and white noise, a similar algorithm with two LMS-type adaptive filters was proposed where the one is to identify the feedback path and the other is associated with estimation of frequencies. For the latter the LMS algorithm for linear prediction is used. The stability analysis of the former was conducted under the assumption that the latter converges to have the desired notch characteristic.

In [3], the stability analysis for simultaneous adaptation of both adaptive filters was taken into account where the incoming signal is an AR process.

In this paper, as another method of frequency estimation we use the constrained LMS algorithm in [4] where the order of the filter is equal to the number of sinusoids. Also, analysis of the coupling effects of the two adaptive filters is conducted for the proposed algorithm. It is shown that the coupling effect disappears near the stationary point. Finally, simulation results show the validity of the theoretical findings.

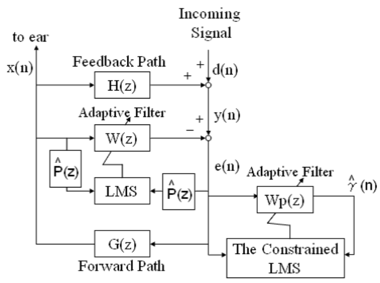

Fig. 1. Block diagram of the proposed algorithm for feedback cancellation

II. The Stationary Point of the Proposed Scheme

Fig.1 shows a block diagram of the proposed scheme. An input signal to the hearing aids is denoted by $d(n)$ and an amplified output signal from the receiver is denoted by $x(n)$. The transfer functions of the forward path (amplifying circuit) and the feedback path are $G(z)$ and $H(z)$, respectively. The forward path transfer function $G(z)$ is the desired characteristic of the hearing aids and is fixed and known, but $H(z)$, the transfer function of feedback path, is unknown and may be slowly time-varying. So, the adaptive filter which is denoted by $W(z)$ is used to identify $H(z)$.

The signals $x(n)$ and $e(n)$ in Fig.1 in the steady state are written by

$$x(n) = G(z)e(n) \tag{1}$$

$$\begin{aligned} e(n) &= y(n) - W(z)x(n) \\ &= d(n) + G(z)(H(z) - W(z))e(n), \end{aligned} \tag{2}$$

and $W(z)$ and $H(z)$ are given by

$$W(z) = w_0 + w_1 z^{-1} + \cdots + w_{L-1} z^{-L+1} \tag{3}$$

$$H(z) = h_0 + h_1 z^{-1} + \cdots + h_{L-1} z^{-L+1} \tag{4}$$

where z^{-1} denotes the unit delay operator and for example, $G(z)e(n)$ means the filtering operation to the signal $e(n)$ by the transfer function $G(z)$. We also see that

$$e(n) = Q(z)d(n),\ Q(z) = \frac{1}{1 + (W(z) - H(z))G(z)} \tag{5}$$

where we assume that $Q(z)$ is stable. As in [2], we use the following modified LMS algorithm for $W(z)$. The tap weight vector $\boldsymbol{w}(n)$ is updated by

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu \boldsymbol{x}'(n)e'(n) \tag{6}$$

where μ is the positive step size, $\boldsymbol{w}(n)$, $\boldsymbol{x}'(n)$ are L dimensional vectors defined by

$$\boldsymbol{w}(n) = [w_0(n), w_1(n), \cdots, w_{L-1}(n)]^T \quad (7)$$

$$\boldsymbol{x}'(n) = [x'(n), x'(n-1), \cdots, x'(n-L+1)]^T \quad (8)$$

and $x'(n)$, $e'(n)$ are defined by

$$x'(n) = \hat{P}(z)x(n), \;\; e'(n) = \hat{P}(z)e(n) \quad (9)$$

where these signals are the filtered version of $x(n)$, $e(n)$ by $\hat{P}(z)$, respectively. The operation $(\cdot)^T$ denotes transpose of matrices or vectors. This $\hat{P}(z)$ needs to be appropriately selected. The stationary point of the adaptive filter was derived by the standard averaging method. The averaged system corresponding to (6) is given by

$$\bar{\boldsymbol{w}}(n+1) = \bar{\boldsymbol{w}}(n) + \mu \mathrm{E}[\boldsymbol{x}'(n)e'(n)]. \quad (10)$$

The stationary point of $\bar{\boldsymbol{w}}(n)$ is obtained by solving

$$\mathrm{E}[\boldsymbol{x}'(n)e'(n)] = \mathbf{0} \quad (11)$$

with respect to $\bar{\boldsymbol{w}}(n) = \bar{\boldsymbol{w}}$. Denoting the PSD (Power Spectral Density) of the incoming signal $d(n)$ as $S(e^{j\omega})$, from (1), (5) and (9) the l-th element of $\mathrm{E}[\boldsymbol{x}'(n)e'(n)]$ is written as

$$(\mathrm{E}[\boldsymbol{x}'(n)e'(n)])_l = \int_{-\pi}^{\pi} \frac{e^{-jl\omega}G(e^{j\omega})}{2\pi} \left| Q(e^{j\omega})\hat{P}(e^{j\omega}) \right|^2 S(e^{j\omega}) \mathrm{d}\omega. \quad (12)$$

We assume that the incoming signal $d(n)$ is a sum of K sinusoids and white noise and express it as

$$d(n) = \sum_{p=1}^{K} A_p e^{j\omega_p n} + \nu(n) \quad (13)$$

where $\nu(n)$ is white noise with mean 0 and variance σ^2. From (13), we have

$$S(e^{j\omega}) = 2\pi \sum_{p=1}^{K} |A_p|^2 \delta(\omega - \omega_p) + \sigma^2. \quad (14)$$

It has been shown in [2] that for sufficiently large L the only solution of (11) is $W_{opt}(z) = H(z)$, if $\hat{P}(z)$ has the notch characteristic

$$\hat{P}(e^{-j\omega_1}) = \hat{P}(e^{-j\omega_2}) = \cdots = \hat{P}(e^{-j\omega_K}) = 0 \quad (15)$$

and the order $q-1$ of $\hat{P}(z)$ satisfies

$$q - 1 \geq K \quad (16)$$

where the forward path transfer function $G(z)$ is set to contain q time unit delays such that

$$G(z) = z^{-q} G_c(z) \quad (17)$$

where $G_c(z)$ is taken to be of minimum phase.

We introduce an adaptive filter $W_p(z)$ for frequency estimation into the system, and update $\hat{P}(z)$. In [2], the LMS algorithm for linear prediction for $e(n)$ is used since at the desired stationary point $e(n) = d(n)$. It has been stated in [2] that if the order of $W_p(z)$ is sufficiently large compared to the number of sinusoids, then desirable unbiased feedback cancellation can be done.

Let us consider what algorithm should be used for $W_p(z)$ in order that at the stationary point it has the notch characteristic with the order $q - 1 = K$. Expressing the autocorrelation matrix of $\boldsymbol{e}(n)$ by R_e where $\boldsymbol{e}(n)$ is defined by

$$\boldsymbol{e}(n) = [e(n), e(n-1), \cdots, e(n-q+1)]^T. \quad (18)$$

When $e(n) = d(n)$, we find that the filter whose coefficients are elements of the eigenvector corresponding to the minimum eigenvalue of R_e has the notch characteristic to the sinusoids. (For details see 4.5 of [5].)

If $\boldsymbol{w_p}$ is the constrained unit-norm weight vector, then minimization of the quadratic form $\boldsymbol{w_p}^T R_e \boldsymbol{w_p}$ leads to the eigenvector corresponding to the minimum eigenvalue of R_e by the method of Lagrange multipliers. The adaptive algorithm is obtained by calculating the gradient of the squared output with respect to the unnormalized weight vector $\boldsymbol{w_r}$, where

$$\boldsymbol{w_p} = \frac{\boldsymbol{w_r}}{\|\boldsymbol{w_r}\|}, \qquad \|\boldsymbol{w_p}\| = 1. \quad (19)$$

The resulting constrained LMS algorithm was derived in [4] as

$$\boldsymbol{w_r}(n+1) = \boldsymbol{w_r}(n) - \mu_p \left[\frac{\boldsymbol{e}(n)\hat{\gamma}(n)}{\|\boldsymbol{w_r}(n)\|} - \frac{\boldsymbol{w_r}(n)\hat{\gamma}(n)^2}{\|\boldsymbol{w_r}(n)\|^2} \right]$$

$$\boldsymbol{w_p}(n) = \frac{\boldsymbol{w_r}(n)}{\|\boldsymbol{w_r}(n)\|} \quad (20)$$

where μ_p is the positive step size. The tap vectors $\boldsymbol{w_p}(n)$, $\boldsymbol{w_r}(n)$ are the estimates of $\boldsymbol{w_p}$, $\boldsymbol{w_r}$ with

$$\boldsymbol{w_p}(n) = [w_{p0}(n), w_{p1}(n), \cdots, w_{p(q-1)}(n)]^T \quad (21)$$

$$\boldsymbol{w_r}(n) = [w_{r0}(n), w_{r1}(n), \cdots, w_{r(q-1)}(n)]^T, \quad (22)$$

and the output error $\hat{\gamma}(n)$ is

$$\hat{\gamma}(n) = w_{p0}(n)e(n) + \cdots + w_{p(q-1)}(n)e(n-q). \quad (23)$$

We set as

$$W_r(z) = w_{r0} + w_{r1}z^{-1} + \cdots + w_{r(q-1)}z^{-q+1}, \quad (24)$$

and $W_p(z)$ can be expressed as $W_p(z) = W_r(z)/\|\boldsymbol{w_r}\|$. Since at the stationary point, the coefficients of $W_r(z)$ correspond to the eignvector associated with the minimum eigenvalue of R_e, we update $\hat{P}(z)$ as

$$\hat{P}(z) = W_r(z). \quad (25)$$

Thus, by using the constrained LMS algorithm for the adaptive filter for frequency estimation, we can make $\hat{P}(z)$ to have the notch characteristic with the order $q-1 \geq K$ at the stationary state.

III. Stability of the Feedback Cancellation Algorithm

Here stability analysis near the desired stationary point is presented when both $W(z)$ and $W_p(z)$ are operating. For (6) and (20) the averaged system is given by

$$\begin{bmatrix} \bar{\boldsymbol{w}}(n+1) \\ \bar{\boldsymbol{w}}_{\boldsymbol{r}}(n+1) \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{w}}(n) \\ \bar{\boldsymbol{w}}_{\boldsymbol{r}}(n) \end{bmatrix} - \mu \mathrm{E} \begin{bmatrix} -\boldsymbol{x}'(n)e'(n) \\ \frac{\boldsymbol{e}(n)\hat{\gamma}(n)}{\|\boldsymbol{w}_{\boldsymbol{r}}(n)\|} - \frac{\boldsymbol{w}_{\boldsymbol{r}}(n)\hat{\gamma}(n)^2}{\|\boldsymbol{w}_{\boldsymbol{r}}(n)\|^2} \end{bmatrix} \tag{26}$$

where for simplicity we set $\mu = \mu_p$. The above discrete-time system can be approximated by the continuous-time linear system near the stationary point as

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \bar{\boldsymbol{w}}(t) \\ \bar{\boldsymbol{w}}_{\boldsymbol{r}}(t) \end{bmatrix} = -\Phi \left(\begin{bmatrix} \bar{\boldsymbol{w}}(t) \\ \bar{\boldsymbol{w}}_{\boldsymbol{r}}(t) \end{bmatrix} - \begin{bmatrix} \boldsymbol{w}_{opt} \\ \boldsymbol{w}_{\boldsymbol{r}opt} \end{bmatrix} \right) \tag{27}$$

where Φ is

$$\begin{bmatrix} \frac{\partial}{\partial \boldsymbol{w}} \mathrm{E}[-\boldsymbol{x}'(n)e'(n)] & \frac{\partial}{\partial \boldsymbol{w}_{\boldsymbol{r}}} \mathrm{E}[-\boldsymbol{x}'(n)e'(n)] \\ \frac{\partial}{\partial \boldsymbol{w}} \mathrm{E}[\frac{\boldsymbol{e}(n)\hat{\gamma}(n)}{\|\boldsymbol{w}_{\boldsymbol{r}}\|} - \frac{\boldsymbol{w}_{\boldsymbol{r}}\hat{\gamma}(n)^2}{\|\boldsymbol{w}_{\boldsymbol{r}}\|^2}] & \frac{\partial}{\partial \boldsymbol{w}_{\boldsymbol{r}}} \mathrm{E}[\frac{\boldsymbol{e}(n)\hat{\gamma}(n)}{\|\boldsymbol{w}_{\boldsymbol{r}}\|} - \frac{\boldsymbol{w}_{\boldsymbol{r}}\hat{\gamma}(n)^2}{\|\boldsymbol{w}_{\boldsymbol{r}}\|^2}] \end{bmatrix} \Bigg|_{\boldsymbol{w}_{opt}, \boldsymbol{w}_{\boldsymbol{r}opt}} \stackrel{\text{def}}{=} \begin{bmatrix} A & B \\ C & D \end{bmatrix}. \tag{28}$$

From Lyapunov stability theory, if $\Phi + \Phi^T$ is positive definite then the adaptive filters are stable near their stationary points. The l-th element of $\mathrm{E}[\boldsymbol{x}'(n)e'(n)]$ is expressed as (12), and the l-th element of $\mathrm{E}[\frac{\boldsymbol{e}(n)\hat{\gamma}(n)}{\|\boldsymbol{w}_{\boldsymbol{r}}\|} - \frac{\boldsymbol{w}_{\boldsymbol{r}}\hat{\gamma}(n)^2}{\|\boldsymbol{w}_{\boldsymbol{r}}\|^2}]$ is

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} |Q(z)|^2 S(z^{-1}) W_r(z) \left(\frac{z^l}{\|\boldsymbol{w}_{\boldsymbol{r}}\|^2} - \frac{w_{rl} W_r(z^{-1})}{\|\boldsymbol{w}_{\boldsymbol{r}}\|^4} \right) \mathrm{d}\omega. \tag{29}$$

At the stationary point, $W(z)$ is $W_{opt}(z) = H(z)$ and from (5) we have

$$Q(z) = 1, \tag{30}$$

and $W_r(z)$, $W_p(z)$ are $W_{r,opt}(z) = \hat{P}_{opt}(z)$, $W_{p,opt}(z) = W_{r,opt}(z)/\|\boldsymbol{w}_{\boldsymbol{r}opt}\|$ respectively where $W_{p,opt}(z)$, $W_{r,opt}(z)$ and $\hat{P}_{opt}(z)$ have the notch characteristic in (15).

We first calculate the block A in the following. The (l,k)-th element of A is

$$A_{lk} = \frac{-\partial}{\partial w_k} \int_{-\pi}^{\pi} \frac{z^l G(z^{-1})}{2\pi} \left| Q(z)\hat{P}(z) \right|^2 S(z^{-1}) \mathrm{d}\omega \Bigg|_{\boldsymbol{w}_{opt}, \boldsymbol{w}_{\boldsymbol{r}opt}} \tag{31}$$

where $l = 0, \cdots, L-1$, and $k = 0, \cdots, L-1$. Partial differentiating and using (14), (17) and (30), we have

$$\begin{aligned} A_{lk} = & \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} z^{l+k+2q} G_c^2(z^{-1}) |\hat{P}_{opt}(z)|^2 \mathrm{d}\omega \\ & + \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} z^{l-k} \left| G_c(z) \right|^2 |\hat{P}_{opt}(z)|^2 \mathrm{d}\omega. \end{aligned} \tag{32}$$

The function $z^{l+k+2q} G_c^2(z^{-1}) |\hat{P}_{opt}(z)|^2$ can be expanded in positive powers of z. So, the first term of RHS of (32) is 0. Hence, A_{lk} is

$$A_{lk} = \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} z^{l-k} \left| G_c(z) \right|^2 |\hat{P}_{opt}(z)|^2 \mathrm{d}\omega. \tag{33}$$

We next calculate the block B in the following. The (l,k)-th element of B is

$$B_{lk} = \frac{-\partial}{\partial w_{rk}} \int_{-\pi}^{\pi} \frac{z^l G(z^{-1})}{2\pi} |Q(z)|^2 S(z^{-1}) |W_r(z)|^2 \mathrm{d}\omega \Bigg|_{\boldsymbol{w}_{opt}, \boldsymbol{w}_{\boldsymbol{r}opt}} \tag{34}$$

where $0 \le l \le L-1$ and $0 \le k \le q-1$. Partial differentiating and using (14), (17) and (30), we have

$$\begin{aligned} B_{lk} = & -\frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} z^{l+q+k} G_c(z^{-1}) W_{r,opt}(z) \mathrm{d}\omega \\ & - \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} z^{l+q-k} G_c(z^{-1}) W_{r,opt}(z^{-1}) \mathrm{d}\omega. \end{aligned} \tag{35}$$

Noting that the order of $W_{r,opt}(z)$ is $q-1$, the function $z^{l+q+k} G_c(z^{-1}) W_{r,opt}(z)$ can be expanded in positive powers of z, and so the first term of RHS is 0. Noting that $k \le q-1$, the function $z^{l+q-k} G_c(z^{-1}) W_{r,opt}(z^{-1})$ can be expanded in positive powers of z, and so the second term of RHS is 0. Hence,

$$B_{lk} = 0. \tag{36}$$

We then calculate the block C in the following. The (l,k)-th element of C is

$$C_{lk} = \frac{\partial}{\partial w_k} \frac{1}{2\pi} \int_{-\pi}^{\pi} |Q(z)|^2 S(z^{-1}) W_r(z) \times \left(\frac{z^l}{\|\boldsymbol{w}_{\boldsymbol{r}}\|^2} - \frac{w_{rl} W_r(z^{-1})}{\|\boldsymbol{w}_{\boldsymbol{r}}\|^4} \right) \mathrm{d}\omega \Bigg|_{\boldsymbol{w}_{opt}, \boldsymbol{w}_{\boldsymbol{r}opt}} \tag{37}$$

where $0 \le l \le q-1$ and $0 \le k \le L-1$. Partial differentiating and using (14), (17) and (30), we have

$$\begin{aligned} C_{lk} = & -\frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} \frac{z^{-k-q+l} G_c(z) W_{r,opt}(z)}{\|\boldsymbol{w}_{\boldsymbol{r}opt}\|^2} \mathrm{d}\omega \\ & + \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} \frac{w_{rl,opt} G_c(z) |W_{r,opt}(z)|^2 z^{-k-q}}{\|\boldsymbol{w}_{\boldsymbol{r}opt}\|^4} \mathrm{d}\omega \\ & - \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} \frac{z^{k+q+l} G_c(z^{-1}) W_{r,opt}(z)}{\|\boldsymbol{w}_{\boldsymbol{r}opt}\|^2} \mathrm{d}\omega \\ & + \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} \frac{w_{rl,opt} G_c(z^{-1}) |W_{r,opt}(z)|^2 z^{k+q}}{\|\boldsymbol{w}_{\boldsymbol{r}opt}\|^4} \mathrm{d}\omega. \end{aligned} \tag{38}$$

Noting that $l \le q-1$ and the order of $W_{r,opt}(z)$ is $q-1$, the functions $z^{-k-q+l} G_c(z) W_{r,opt}(z)$ and $G_c(z) |W_{r,opt}(z)|^2 z^{-k-q}$ can be expanded in negative powers of z, and so the first and second terms of RHS are 0. Noting that the order of $W_{r,opt}(z)$ is $q-1$, the functions $z^{k+q+l} G_c(z^{-1}) W_{r,opt}(z)$ and $G_c(z^{-1}) |W_{r,opt}(z)|^2 z^{k+q}$ can be expanded in positive powers of z, and so the third and fourth terms of RHS are 0. Hence,

$$C_{lk} = 0. \tag{39}$$

From (36) and (39) it is seen that two adaptive filters are decoupling near the stationary point.

We finally calculate the block D in the following. The (l, k)-th element of D is

$$D_{lk} = \frac{\partial}{\partial w_{rk}} \frac{1}{2\pi} \int_{-\pi}^{\pi} |Q(z)|^2 S(z^{-1}) W_r(z) \times \left(\frac{z^l}{\|\boldsymbol{w_r}\|^2} - \frac{w_{rl} W_r(z^{-1})}{\|\boldsymbol{w_r}\|^4} \right) \mathrm{d}\omega \Bigg|_{\boldsymbol{w}_{opt}, \boldsymbol{w_r}_{opt}} \quad (40)$$

where $0 \le l \le q-1$ and $0 \le k \le q-1$. Partial differentiating and using (14), (17) and (30), we have

$$\begin{aligned} D_{lk} =& \sum_{p=1}^{K} |A_p|^2 \frac{e^{j\omega_p (k-l)}}{\|\boldsymbol{w_r}_{opt}\|^2} \\ &+ \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} z^l \left(\frac{-2 w_{rk,opt} W_{p,opt}(z)}{\|\boldsymbol{w_r}_{opt}\|^3} + \frac{z^{-k}}{\|\boldsymbol{w_r}_{opt}\|^2} \right) \mathrm{d}\omega \\ &- \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} \Bigg\{ \left(\frac{\delta(l-k)}{\|\boldsymbol{w_r}_{opt}\|^2} - \frac{4 w_{rl,opt} w_{rk,opt}}{\|\boldsymbol{w_r}_{opt}\|^4} \right) |W_{p,opt}(z)|^2 + \\ &\frac{w_{rl,opt}}{\|\boldsymbol{w_r}_{opt}\|^3} \left(W_{p,opt}(z^{-1}) z^{-k} + W_{p,opt}(z) z^k \right) \Bigg\} \mathrm{d}\omega. \quad (41) \end{aligned}$$

By calculating the integrals in (41) and using (19), we see that the second and third terms cancell. Hence D_{lk} becomes

$$D_{lk} = \sum_{p=1}^{K} |A_p|^2 \frac{e^{j\omega_p (k-l)}}{\|\boldsymbol{w_r}_{opt}\|^2}. \quad (42)$$

Hence, from (33), (36), (39) and (42) the Hermitian form of Φ is

$$\left[\begin{array}{cc} \boldsymbol{\xi}^H & \boldsymbol{\eta}^H \end{array} \right] \Phi \left[\begin{array}{c} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{array} \right] = \frac{\sigma^2}{2\pi} \int_{-\pi}^{\pi} \left(\left| \sum_{i=0}^{L-1} \xi_i z^{-i} \right|^2 |G_c(z)|^2 \times |\hat{P}_{opt}(z)|^2 \right) \mathrm{d}\omega + \sum_{p=1}^{K} \frac{|A_p|^2}{\|\boldsymbol{w_r}_{opt}\|^2} \left| \sum_{i=0}^{q-1} \eta_i e^{j\omega_p i} \right|^2. \quad (43)$$

The block A for $W(z)$ is positive definite, so $W(z)$ is stable as in [2]. But, the block D for $W_r(z)$ is positive semidefinite. From the second term of RHS of (43) we see that all vectors $\boldsymbol{\eta} \neq \mathbf{0}$ such that $\boldsymbol{\eta}^H D \boldsymbol{\eta} = 0$ correspond to the coefficient vector satisfying (15). This means that the adaptive filter $W_p(z)$ has the desired notch characteristic whenever it stops. Hence, the proposed algorithm is stable for the desirable characteristic although Φ is positive semidefinite.

IV. Simulation Results

To see the validities of the theoretical findings in previous sections, some simulation results are presented.

First, we present results for sinusoidal signals. We use the following misalignment as the performance index $\zeta_h = \sum_{i=0}^{L-1} (h_i - w_i)^2 / \sum_{i=0}^{L-1} {h_i}^2$. The feedback and forward path transfer functions are set to $H(z) = 0.2z^{-1}$ and $G(z) = 2.0z^{-q}$ respectively. For the adaptive filter $W(z)$ we set the tap weight length to $L = 2$ and the step size to $\mu = 1.0 \times 10^{-3}$. The adaptive filter $W_p(z)$ is $(q-1)$-th order and its step size is $\mu_p = 1.0 \times 10^{-2}$. Referring to (13), $\nu(n)$ is Gaussian i.i.d, with zero mean and variance $\sigma^2 = 0.01$. In Fig.2 it is a sum of two sinusoids with $K = 4$, $A_1 = A_2 = A_3 = A_4 = 1/2$, $\omega_1, \omega_2 = \pm\pi/4$, $\omega_3, \omega_4 = \pm\pi/7$, and q is set to 5. When the LMS algorithm for linear prediction is used, the misalignment does not converge at all. As for the constrained LMS algorithm, the desired convergence is obtained.

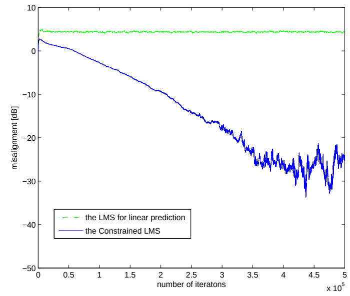

Fig. 2. Plots of misalignment ζ_h in the case where $K = 4$ and $q = 5$.

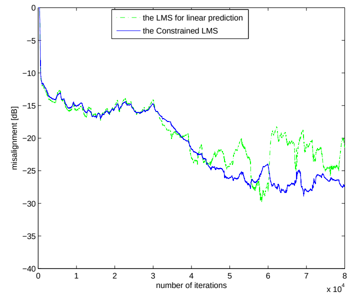

Fig. 3. Plots of misalignment ζ_h in the case where the input is actual music signal and $q = 100$.

Second, we present results for actual music signal. Fig.3 shows the results where $L = 5$, $G_c(z) = 3$, and $q = 100$ with $\mu_p = 10^{-2}$ and $\mu = 10^{-2}$. It is seen that the algorithm with the constrained LMS performs a little bit better than the LMS for linear prediction.

V. Conclusion

In this paper we have presented convergence analysis of the modified LMS-type algorithm for hearing aids with sinusoidal input signal. It has been shown that the adaptive filters have the desired stationary points and the stabilities near the points.

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