Design of Oversampled Cosine-Sine Modulated Filter Banks for Directional Image Representation

Ryutaro Ogawa*, Seisuke Kyochi*, and Taizo Suzuki†
* Dept. of Information and Media Engineering, The University of Kitakyushu, Fukuoka, Japan
E-mail: v4mca004@eng.kitakyu-u.ac.jp, s-kyochi@kitakyu-u.ac.jp
† Faculty of Engineering, Information and Systems, University of Tsukuba, Ibaraki, Japan
E-mail: taizo@cs.tsukuba.ac.jp

***Abstract*—We propose an oversampled cosine-sine modulated filter bank (OCSMFB) for richer overcompleteness contributing sparse directional image representation. The CSMFB, proposed as an alternative of the dual-tree complex wavelet transform (DTCWT), can be designed easily by modulating a prototype filter and can work in image processing tasks better than the DTCWT. However, the conventional CSMFB consists of two critically-sampled FB, and its non-redundancy restricts design degree of freedom and overcomplete image representation. This paper relaxes the redundancy of the conventional CSMFB by oversampling each CMFB and SMFB. For design of the OCSMFB, we present the perfect reconstruction (PR) condition and the parameterization, in order to satisfy the PR structurally. Moreover, the parameterization for structural regularity, which provides NO DC leakage, is discussed. The experimental results of image denoising show the OCSMFB can work better than the CSMFB.**

I. Introduction

For decades, extended discrete wavelet transforms satisfying "rich directional selectivity[1]" (DS) have been widely studied, such as *dual-tree complex wavelet transforms* (DTCWTs) [1], [2], *Contourlet* [3], *Curvelet* [4], and so on. Among them, DTCWTs have been paid much attenuation due to their low computational complexity [1], [2]. While directional selective transforms usually require high computational complexity, such as two dimensional (2D) non-separable filtering, DTCWTs can save computational complexity by 2D separable implementation of parallel filer banks (FBs), i.e., cascading vertical and horizontal filtering. Furthermore, they also satisfy "shift-invariance" (SI). Thanks to DS and SI, it can be successfully applied to various kinds of practical image processing, such as image denoising, image analysis, and image compression [1].

As well as the DTCWT, another DS and SI transforms, cosine-sine modulated filter banks (CSMFBs), have been shown recently [5], [6], [7]. M-channel DTCWTs are difficult to design with good filter performances, e.g., stopband attenuation or coding gain, in time-domain, due to the fractional delay requirement imposed on parallel FBs, which degrades design degree of freedom. On the other hand, because CSMFBs require NO fractional delay requirement (just the modulation of a prototype filter), M-channel CSMFBs can be better performance than M-channel DTCWTs and effectively work in applications.

One problem on the conventional CSMFBs is the limitation of redundancy, i.e., both CMFB and SMFB are critically-sampled FBs and the resulting redundant ratio is 2. This poor redundancy degrades the design degree of freedom. Since the design degree of freedom of the CSMFB is only in a prototype filter, filter performance cannot be improved under short filter length. Moreover, rich redundancy is generally desired to enhance the performance in many image processing tasks, where redundant image representation is allowed.

This paper attempts to relax the limitation of critical sampling to oversampling and realizes oversampled CSMFBs (OCSMFBs). Note that the oversampled CMFB (OCMFB) has already been proposed in [8], whereas the oversampled SMFB (OSMFB) has not yet. Thus the perfect reconstruction (PR) condition for the OSMFB is firstly discussed in this paper. Moreover, the parameterization of the OCSMFB for the PR is proposed. In addition, the parameterization for structural regularity, which is important for NO DC leakage [9], is also shown. The designed OCSMFB is applied to image denoising as a practical application. Experimental results show that the OCSMFB outperforms the CSMFB with respect to both of the reconstruction error and the visual quality.

The rest of this paper is organized as follows. Sec. II briefly reviews about the CSMFB and the OCMFB. The PR condition for OSMFB is discussed in Sec. III-A. Then, the parameterizations for PR and structural regularity are given in Sec. III-B and Sec. III-C, respectively. Sec. IV evaluates the OCSMFB in image denoising. Finally, this paper is concluded in Sec. V.

Notations: $H(z)$ is defined as $H(z) := \sum_n h(n)z^{-n}$ and $\widetilde{H}(z) := H^*(z^{-1})$. The $M \times L$ FB means the M-channel filter bank with the filter length of L. $\mathbf{I}_N$, $\mathbf{J}_N$, and $\mathrm{diag}(a_0, \ldots, a_{N-1})$ are the identity, the reversal identity, and the diagonal matrices, respectively.

II. Preliminaries

A. Cosine-Sine Modulated Filter Banks and Oversampled Cosine Modulated Filter Banks

A CSMFB consists of two maximally decimated FBs as in Fig. 1 ($D = M$). Its filter coefficients of the analysis subband

[1]Directional selectivity means sparse representation capability for directionally oriented components of images, e.g., lines and edges. For that, basis functions should be oriented along various kinds of directions [1].

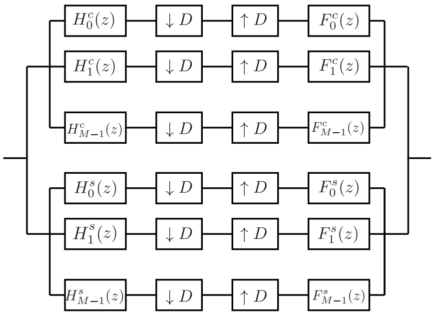


Fig. 1. M-channel CSMFB ($D = M$). Upper FB: CMFB, Lower FB: SMFB.

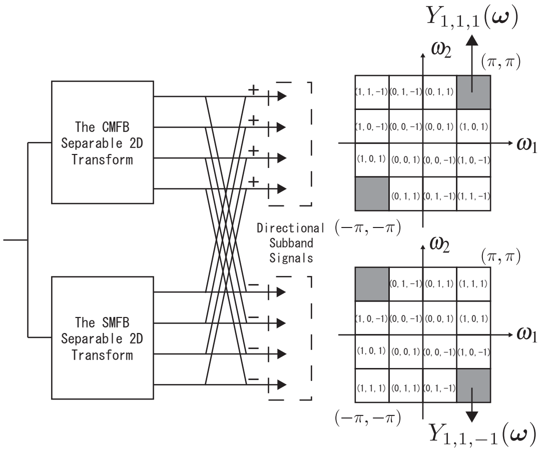


Fig. 2. Procedure of the directional image decomposition via M-channel CSMFB.

filters $\{H_k^c(z), H_k^s(z)\}$ are expressed as follows [5]:

$$\begin{cases} h_k^c(n) := 2p(n)\cos\left((k+\frac{1}{2})\frac{\pi}{M}\left(n - \frac{N-1}{2}\right) + \theta_k\right) \\ h_k^s(n) := 2p(n)\sin\left((k+\frac{1}{2})\frac{\pi}{M}\left(n - \frac{N-1}{2}\right) + \theta_k\right), \end{cases} \tag{1}$$

where $k = 0, \ldots, M-1$, $N = 2mM - 1$, $\theta_k = (-1)^k \frac{\pi}{4}$, and $p(n)$ is the prototype filter. In this paper, we assume the paraunitariness on FBs. Thus, the synthesis subband filters are the time-reversed versions of the analysis filters, i.e., $f_k^c(n) = h_k^c(N-1-n)$ and $f_k^s(n) = h_k^s(N-1-n)$.

The PR condition is imposed on the prototype filter. Let $G_k(z)$ $(k = 0, \ldots, 2M-1)$ be the polyphase components of the prototype filter $P(z)$, i.e., $P(z) = \sum G_k(z)z^{-k}$, $G_k(z) = \sum p(2Mn+k)z^{-q}$. The PR property for CSMFB can be expressed as:

$$\widetilde{G}_k(z)G_k(z) + \widetilde{G}_{M+k}(z)G_{M+k}(z) = \frac{1}{2M}, \tag{2}$$

where $k = 0, \ldots, \frac{M}{2} - 1$.

In our previous work [5], the directionality of CSMFBs is derived according to the relationship of cosine and sine modulation between two FBs. For the directional image representation, the original image is firstly decomposed by two separable 2D transforms of the CMFB and the SMFB. Then, the directional coefficients are obtained by addition and subtraction of the CMFB and the SMFB coefficients. The procedure is illustrated in Fig. 2.

On the other hand, Bölcskei et al. have discussed the PR condition for OCMFB (as $D = \frac{M}{2^L}$ in Fig. 1) [8]

$$\sum_{\ell=0}^{4L-1} \widetilde{G}_{k+D\ell}(z)G_{k+D\ell}(z) = \frac{1}{2M}, \tag{3}$$

where $k = 0, \ldots, \frac{D}{2} - 1$.

III. Oversampled Cosine-Sine Modulated Filter Banks

A. Perfect Reconstruction Condition for OSMFB

This section gives the PR condition of the OSMFB, which has not been discussed yet, for constructing OCSMFB. Consider the same definition of the subband filters for SMFB (1), but the decimation factor is $D = \frac{M}{2^L}$ $(1 \leq L \leq \log_2 M - 1)$. In the paraunitary OCSMFB, $h_k^{(2)}(n) = (-1)^k h_k^{(1)}(N-1-n)$. From the time-domain relationship between the OCMFB and the OSMFB, the relationship between the polyphase matrix of the OCMFB $\mathbf{E}^c(z)$ and the OSMFB $\mathbf{E}^s(z)$ can be expressed as:

$$\mathbf{E}^S(z^D) = z^{-(N-(D-1))}\mathbf{\Gamma}\mathbf{E}^C(z^{-D})\mathbf{J}_D, \tag{4}$$

where $\mathbf{\Gamma} = \mathrm{diag}(1, -1, \ldots, 1, -1)$. Assume that the OCMFB satisfies the PR condition $\widetilde{\mathbf{E}}^C(z)\mathbf{E}^C(z) = \mathbf{I}_D$. By substituting (4) into $\widetilde{\mathbf{E}}^S(z^D)\mathbf{E}^S(z^D)$, it can be reduced that

$$\widetilde{\mathbf{E}}^S(z^D)\mathbf{E}^S(z^D) = \mathbf{I}_D. \tag{5}$$

If the OCMFB satisfies the PR, the OSMFB also satisfies the PR.

B. Parameterization for Perfect Reconstruction

The parameterization for the CMFB (and SMFB) was presented in [10]. It is briefly reviewed in the following. For simple discussion, the numbers of the channel and the decimation factor are 4. Then, the PR condition on the 4-channel and 4-decimated CMFB/SMFB can be expressed as:

$$\widetilde{G}_{2k}(z)G_{2k}(z) + \widetilde{G}_{2k+4}(z)G_{2k+4}(z) = \frac{1}{8}, \ (k = 0, 1). \tag{6}$$

For given filter length $N = 2mM$, the parameterization for the above equation can be given as [10]:

$$\begin{bmatrix} G_{2k}(z) \\ G_{2k+4}(z) \end{bmatrix} = \prod_{\ell=m-1}^{1} \left\{ \mathbf{V}_{k,\ell} \begin{bmatrix} 1 & \\ & z^{-1} \end{bmatrix} \right\} \begin{bmatrix} \cos\theta_k \\ \sin\theta_k \end{bmatrix}, \tag{7}$$

where $\mathbf{V}_{k,\ell}$ is a rotation matrix. The total number of the design parameter is $2m$.

On the other hand, according to (3), the PR condition for the 4-channel and 2-decimated OCSMFB is reduced as:

$$\sum_{k=0}^{3} \widetilde{G}_{2k}(z)G_{2k}(z) = \frac{1}{8}. \tag{8}$$

Note that the parameterization of (8) can be realized by applying (7) to $\{G_0(z),\ G_4(z)\}$ and $\{G_2(z),\ G_6(z)\}$, independently, which is presented in [8]. However, this cannot fully exploit the design degree of freedom. This paper introduces a wider class of the parameterization as follows:

$$\begin{bmatrix} G_0(z) \\ G_2(z) \\ G_4(z) \\ G_6(z) \end{bmatrix} = \prod_{k=m-1}^{1} \left\{ \mathbf{V}_{0,k} \begin{bmatrix} \mathbf{I}_2 & \\ & z^{-1}\mathbf{I}_2 \end{bmatrix} \right\} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad , \tag{9}$$

where $\mathbf{V}_{0,k}$ is a 4×4 orthogonal matrix and $\{a_i\}_{i=0}^{3}$ are real numbers satisfying $\sum_{i=0}^{3} a_i^2 = 1$. Generally, an $M \times M$ orthogonal matrix $\mathbf{V}$ can be characterized by the product of the $\frac{M(M-1)}{2}$ givens rotation matrices as [11]: and $\{a_i\}_{i=0}^{M-1}$ satisfying $\sum_{i=0}^{M-1} a_i^2 = 1$ are parameterized by $M-1$ angles as:

$$\begin{cases} a_0 = \cos\theta_0 \\ a_k = \left(\prod_{i=0}^{k-1} \sin\theta_i \right) \cos\theta_k, \ (k = 1, \ldots, M-1) \end{cases} . \tag{10}$$

Since $M = 4$ in this case, the number of the design parameter is $6(m-1)+3$. Clearly, the proposed parameterization covers the wider class of the OCSMFB and provides richer design parameters. The above discussion can be similarly applied to the general M-channel D-decimated OCSMFB parameterization.

C. Parameterization for Structural Regularity

The regularity condition is important for many image processing tasks [9]. Lack of regularity causes DC leakage that sometimes degrades the energy compaction of natural images, and thus efficient sparse representation cannot be achieved. In [12], the parameterization of the CMFB for structural regularity has been discussed. This section extends it to the OCMFB/OSMFB. As well as the previous section, we consider 4-channel and 2-decimated OCSMFB. First of all, the definition on the regularity is described as:

$$\begin{bmatrix} H_0(1) \\ H_1(1) \\ \vdots \\ H_{M-1}(1) \end{bmatrix} = \mathbf{E}(z^D) \left. \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(D-1)} \end{bmatrix} \right|_{z=1} = \begin{bmatrix} c \\ 0 \\ \vdots \\ 0 \end{bmatrix} . \tag{11}$$

According to (1), the polyphase matrix of the OCMFB $\mathbf{E}(z^D)$ is given as:

$$\mathbf{E}(z^2) = \mathbf{C_a} \begin{bmatrix} \mathbf{g}_0(-z^4) \\ z^{-2}\mathbf{g}_1(-z^4) \\ z^{-4}\mathbf{g}_2(-z^4) \\ z^{-6}\mathbf{g}_3(-z^4) \end{bmatrix} , \tag{12}$$

$$\mathbf{C_a} = \begin{cases} \sqrt{M}(-1)^{m/2}\mathbf{C}_{IV} \left[\, (\mathbf{I}-\mathbf{J}) \quad -(\mathbf{I}+\mathbf{J}) \, \right] \\ \qquad \text{for } m \text{ even} \\ \sqrt{M}(-1)^{(m-1)/2}\mathbf{C}_{IV} \left[\, (\mathbf{I}+\mathbf{J}) \quad (\mathbf{I}-\mathbf{J}) \, \right] \\ \qquad \text{for } m \text{ odd} \end{cases} ,$$

$$\mathbf{g}_0(z) \triangleq \text{diag}\left[G_0(z), G_1(z)\right], \ \mathbf{g}_1(z) \triangleq \text{diag}\left[G_2(z), G_3(z)\right],$$
$$\mathbf{g}_2(z) \triangleq \text{diag}\left[G_4(z), G_5(z)\right], \ \mathbf{g}_3(z) \triangleq \text{diag}\left[G_6(z), G_7(z)\right],$$

where $\mathbf{C}_{IV}$ indicates type-IV DCT. By substituting (12) into (11) as $z = 1$, it can be derived as:

$$\frac{2}{\sqrt{2M}} \begin{bmatrix} G_0(-1) \\ G_2(-1) \\ G_4(-1) \\ G_6(-1) \end{bmatrix} = \frac{c}{\sqrt{M}} \begin{bmatrix} d_{0,3} \\ d_{0,1} \\ d_{0,0} \\ d_{0,2} \end{bmatrix} . \tag{13}$$

Here, if $m = 1$, three angles θ_0, θ_1, and θ_2 for $[a_0, a_1, a_2, a_3]^T$ can be uniquely determined as:

$$\begin{cases} \theta_2 = \tan^{-1}\left(\frac{d_{0,2}}{d_{0,0}}\right) \\ \theta_1 = \tan^{-1}\left(\frac{1}{\cos\theta_2}\frac{d_{0,0}}{d_{0,1}}\right) . \\ \theta_0 = \tan^{-1}\left(\frac{1}{\cos\theta_1}\frac{d_{0,1}}{d_{0,3}}\right) \end{cases} \tag{14}$$

For general order m, (13) can be reduced as:

$$\begin{bmatrix} G_0(-1) \\ G_2(-1) \\ G_4(-1) \\ G_6(-1) \end{bmatrix} = \begin{cases} \frac{1}{(-1)^{m/2}}\mathbf{\Delta}\mathbf{d}_0 \ (m \text{ even}) \\ \frac{1}{(-1)^{(m-1)/2}}\mathbf{d}_1 \ (m \text{ odd}) \end{cases} , \tag{15}$$

where $\mathbf{d}_0 = [d_{0,0}, d_{0,2}, d_{0,3}, d_{0,1}]^T$, $\mathbf{d}_1 = [d_{0,3}, d_{0,1}, d_{0,0}, d_{0,2}]^T$, $\mathbf{\Delta} = \text{diag}(1, 1, -1, -1)$. From (15), it is enough to impose the condition on only the matrix $\mathbf{V}_{0,m-1}$ which can be expressed as:

$$\mathbf{V}_{0,m-1} = \mathbf{I} - 2\gamma\gamma^T$$
$$\gamma = \frac{\mathbf{d}_i - \left(\prod_{k=m-2}^{1} \mathbf{V}_{0,k}\mathbf{a}\right)}{\left\| \mathbf{d}_i - \left(\prod_{k=m-2}^{1} \mathbf{V}_{0,k}\mathbf{a}\right) \right\|_2}, \ (i = 0, \ 1). \tag{16}$$

The parameterization of structural regularity for general M-channel D-decimated OCSMFBs can be given, similarly.

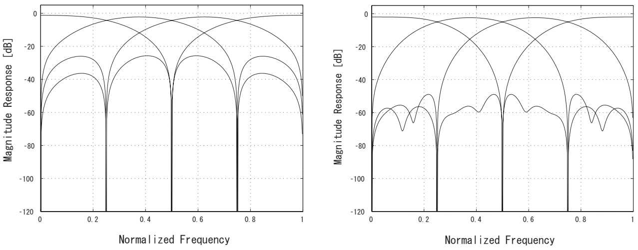
IV. Experimental Results

A. Design Examples

We designed the prototype filters of the 4-channel CSMFB and 4-channel 2-decimated OCSMFB with the filter length of 16 ($m = 2$) and the regularity. The regularity for the conventional CSMFB was imposed by using the method presented in [12]. The numbers of the design parameters are 2 and 3 for the CSMFB and the OCSMFB, respectively. The parameters of the prototype filter $P(z)$ were optimized to minimize stopband attenuation: $C = \int_{\frac{\pi}{2M}}^{\pi} |P(\omega)| d\omega$. The designed frequency responses are shown in Fig. 3. Clearly, the frequency responses of the OCSMFB satisfy the regularity condition and achieve better stopband attenuation.

B. Image Denoising

In this simulation, image denoising was demonstrated to verify the performance of OCSMFB. The Gaussian random noise with variance σ^2 ($\sigma = 30$) is added to the test images shown in Fig. 4. The 4×8, 4×16, 8×16, and 8×32 CSMFBs and those of OCSMFBs with the decimation factor of 2 were newly designed by minimizing the stopband attenuation. We



(a) 4×16 CSMFB (b) 4×16 OCSMFB

Fig. 3. Examples of frequency responses.

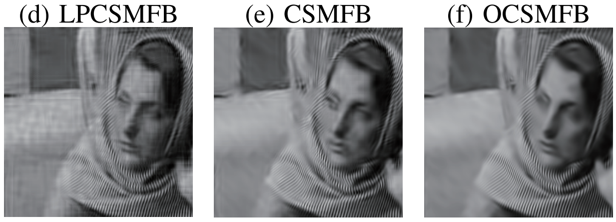
(a) (b) (c) (d)

Fig. 4. Original images: (a) *Zone plate*, (b) *Lena*, (c) *Barbara*, (d) *Pepper*.

also used the 4×10, 4×16, 8×22, and 8×36 DTCWTs designed by [2], and the 5×8, 5×16, 9×16, and 9×32 linear phase CSMFBs (LPCSMFBs) [7] for comparison. The noisy images were firstly applied by 1-level decomposition of each transform, then performed by hard-thresholding with $Th = 3\sigma$ [4]. We used PSNR for the numerical metric of the denoising performance. As shown in Table I and Fig. 5, the OCSMFB shows better numerical and visual quality than those of the CSMFBs, the DTCWTs, and the LPCSMFBs, in most cases.

(a) Noisy (b) CSMFB (c) OCSMFB

(d) LPCSMFB (e) CSMFB (f) OCSMFB



(g) LPCSMFB (h) CSMFB (i) OCSMFB

Fig. 5. (a): Noisy image (*Barbara*), (b), (c): Denoised images, and (d)-(f): Zoomed denoised images. LPCSMFB: 9×32, CSMFB/OCSMFB: 8×32.

TABLE I
THE NUMERICAL RESULTS OF IMAGE DENOISING (PSNR [DB])

Test image	*Zoneplate*	*Lena*	*Barbara*	*Pepper*
Noisy image	18.59	18.59	18.59	18.59
4×10 DTCWT [2]	20.02	27.76	24.53	27.65
4×8 CSMFB [5]	22.64	28.28	25.29	28.07
5×8 LPCSMFB [7]	21.21	27.18	25.54	27.31
4×8 OCSMFB	**24.46**	**28.96**	**25.94**	**28.81**
4×16 DTCWT [2]	21.04	27.90	24.72	27.79
4×16 CSMFB [5]	23.90	28.39	25.63	27.90
5×16 LPCSMFB [7]	23.32	28.30	**26.29**	28.26
4×16 OCSMFB	**25.01**	**28.92**	26.15	**28.67**
8×22 DTCWT [2]	20.92	27.43	25.41	27.63
8×16 CSMFB [5]	25.81	28.41	26.76	28.32
9×16 LPCSMFB [7]	22.68	27.12	25.99	27.13
8×16 OCSMFB	**27.85**	**29.32**	**27.75**	**29.10**
8×36 DTCWT [2]	21.98	27.77	25.81	27.70
8×32 CSMFB [5]	25.35	28.46	26.87	28.05
9×32 LPCSMFB [7]	25.06	27.87	26.75	27.68
8×32 OCSMFB	**28.01**	**29.27**	**27.80**	**29.12**

V. CONCLUSION

In this paper we proposed the OCSMFB for better design degree of freedom and richer overcompleteness. In order to construct the OCSMFB, we firstly showed the PR condition for the OSMFB, which has not been discussed yet. Then, we gave the design parameterization for the PR and the structural regularity. Finally, in the image denoising simulation, it was clarified that the proposed OCSMFB provides better numerical and visual reconstruction quality compared with the conventional CSMFBs and the DTCWTs.

REFERENCES

[1] R. G. Baraniuk I. W. Selesnick and N. G. Kingsbury, "The dual-tree complex wavelet transform," *IEEE Signal Process. Mag.*, pp. 123–151, Nov. 2005.

[2] I. Bayram and I.W. Selesnick, "On the dual-tree complex wavelet packet and M-band transforms," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2298–2310, June 2008.

[3] M. N. Do and M. Vetterli, "The contourlet transform : An efficient directional multiresolution image representation," *IEEE Trans. Image Process.*, vol. 14, no. 12, pp. 2091–2106, 2005.

[4] E. J. Candès and D. L. Donoho, "New tight frames of curvelets and optimal representations of objects with piecewise C^2 singularities," *Comm. Pure Appl. Math*, vol. 56, no. 2, pp. 219–266, Feb. 2004.

[5] S. Kyochi, T. Uto, and M. Ikehara, "Dual-tree complex wavelet transform arising from cosine-sine modulated filter banks," in *Proc. IEEE Int. Symp. Circuits Syst.*, May 2009, pp. 2189–2192.

[6] S. Kyochi and M. Ikehara, "A class of near shift-invariant and orientation-selective transform based on delay-less oversampled even-stacked cosine-modulated filter banks," *IEICE Trans. Fundam. Electron, Commun. Comput,*, vol. E93-A, pp. 724–733, Apr. 2010.

[7] L. Liang and H. Liu, "Dual-tree cosine-modulated filter bank with linear-phase individual filters: An alternative shift-invariant and directional-selective transform," *IEEE Trans. Image Process.*, vol. 22, no. 12, pp. 5168–5180, Dec. 2013.

[8] H. Bölcskei and F. Hlawatsch, "Oversampled cosine modulated filter banks with perfect reconstruction," *IEEE Trans. Circuits and Syst. II*, vol. 45, no. 8, pp. 1057–1071, Aug. 1998.

[9] G. Strang and T. Q. Nguyen, *Wavelets and filter banks*, Wellesley-Cambridge Press, 1997.

[10] R. D. Koilpillai and P. P. Vaidyanathan, "Cosine-modulated FIR filter banks satisfying perfect reconstruction," *IEEE Trans. Signal Process.*, vol. 40, no. 4, pp. 770–783, Apr. 1992.

[11] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Prentice Hall, Englewood Cliffs, 1992.

[12] M. Ikehara T. Uto and K. Ohue, "A novel design of regular cosine-modulated filter banks for image coding," *IEICE Trans. on Fundamentals*, vol. E92-A, no. 7, pp. 1633–1641, Jul. 2009.