A Reduced-dimensional Music Algorithm with Modulus Constraint Based on Electromagnetic Vector Sensor Array

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***Abstract*—To reduce the large computation of joint direction-of-arrival (DOA) and polarization parameters estimation based on electromagnetic vector sensor array, a reduced-dimensional MUSIC algorithm with modulus constraint (MC-RDMUSIC) is proposed. The MC-RDMUSIC algorithm estimates the DOA and polarization parameters in two separate steps. In step one, DOA parameter is separated from polarization parameters and estimated by MUSIC algorithm; in step two, cost function is constructed according to constraint condition and polarization parameters are obtained with closed-form formulas. Compared with traditional MUSIC algorithm, MC-RDMUSIC algorithm avoids the spectrum peak search of DOA and polarization parameters at the same time, greatly improves the operation efficiency. Because of the one-to-one correspondence between DOA angle and polarization parameters, MC-RDMUSIC algorithm avoids parameters matching problem compared with ESPRIT algorithm. The computer simulation results verify the correctness of MC-RDMUSIC algorithm, and as well as its high success rate and precision of the angle estimation.**

Index Terms— Electromagnetic vector sensor, parameter estimation, MUSIC algorithm, spectrum peak search

I. Introduction

Electromagnetic vector sensor array signal processing has become a new research hotspot in the field of array signal processing, and gotten some theoretical results [1].High resolution parameter estimation methods, such as MUSIC [2-3] and ESPRIT algorithm [4-5] have been proposed with good estimation and identify performance for uncorrelated multiple objects. However, for multiple parameters estimation, traditional MUSIC algorithm requires a multi-dimensional spectrum peak search, which makes the calculation complexity seriously increased. At the same time, ESPRIT algorithm requires considering parameters matching problem. Reference [6-8] used reduced-dimensional MUSIC algorithm for scalar array, but no research about electromagnetic vector sensor array. Reference [9-10] used reduced-dimensional MUSIC algorithm based on quaternion and searched DOA angles and polarization parameter respectively, and one four-dimensional spectrum peak search reduced two two-dimensional spectral peak searches. On the basis of literature [9-10], this paper proposes a reduced-dimensional MUSIC algorithm with modulus constraint, which splits signal arrival angles and polarization parameters from each other and estimates them separately. In this way, the four-dimensional spectrum peak search process is reduced to a two-dimensional spectral peak search process. Compared with the algorithm proposed in the literature [10], computational complexity greatly reduces again, and also the DOA and polarization parameters estimation can be accurately obtained.

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II. Array Signal Model

The electromagnetic vector sensor array is made up of M electric crossed dipole pairs. All electric crossed dipole pairs are arranged uniformly in the y axis and deployed paralleled to x axis and y axis respectively. The inter-element spacing is equal to d .

The two orthogonal electric dipole polarization steering vector is expressed as

$$\boldsymbol{a}_p(\theta,\varphi,\gamma,\eta) = \boldsymbol{V}(\theta,\varphi)\cdot\boldsymbol{E}(\gamma,\eta) = \begin{bmatrix} -\sin\varphi & \cos\theta\cos\varphi \\ \cos\varphi & \cos\theta\sin\varphi \end{bmatrix}\begin{bmatrix} \cos\gamma \\ \sin\gamma e^{j\eta} \end{bmatrix} \tag{1}$$

Where $\gamma\in[0,\pi/2)$ denotes auxiliary polarization angle, $\eta\in[-\pi,\pi)$ denotes polarization phase difference. $\theta\in[-\pi/2,\pi/2]$ and $\varphi\in[0,2\pi)$ denote the incident source's elevation angle and azimuth angle respectively.

The space steering vector of the non-collocated array is

$$\boldsymbol{a}_s(\theta,\varphi) = \left[1, e^{-j\phi},\ldots,e^{-j(M-1)\phi}\right]^{\mathrm{T}} \tag{2}$$

Where $\phi = 2\pi d\sin\theta\sin\varphi/\lambda$, λ denotes wavelength.

The space-polarization steering vector can be described as Kronecker product of space steering vector and polarization steering vector, that is

$$\boldsymbol{a}(\theta,\varphi,\gamma,\eta) = \boldsymbol{a}_s(\theta,\varphi)\otimes\boldsymbol{a}_p(\theta,\varphi,\gamma,\eta) \tag{3}$$

Where $\otimes$ denotes the Kronecker product.

The assumed signal scenario has K uncorrelated narrow-band sources which incident on electromagnetic vector sensor array. The received signal model of array can be expressed as

$$\begin{aligned} \boldsymbol{X}(t) &= [x_{1x}, x_{1y}, \ldots, x_{ix}, x_{iy}, \ldots, x_{Mx}, x_{My}]^{\mathrm{T}} \\ &= \sum_{i=1}^{K} \boldsymbol{a}_i s_i(t) + \boldsymbol{n}_i(t) \\ &= \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \end{aligned} \tag{4}$$

Where $\boldsymbol{A} = [\boldsymbol{a}_1, \ldots, \boldsymbol{a}_i, \ldots, \boldsymbol{a}_K]$ denotes array manifold matrix; $\boldsymbol{a}_i$ denotes space-polarization steering vector; $\boldsymbol{s}(t)$ denotes signal vector; $\boldsymbol{n}(t)$ symbolizes the zero-mean additive complex Gaussian noise.

III. Reduced-dimensional MUSIC Algorithm With Modulus Constraint

The array covariance matrix has the form

$$\boldsymbol{R}_x = E\left[\boldsymbol{X}(t)\boldsymbol{X}(t)^{\mathrm{H}}\right] = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^{\mathrm{H}} + \sigma^2 \boldsymbol{I}_{2M} \tag{5}$$

The eigenvalue decomposition of array covariance matrix is

$$\boldsymbol{R}_x = \sum_{i=1}^{2M} \lambda_i \boldsymbol{u}_i \boldsymbol{u}_i^{\mathrm{H}} \tag{6}$$

Where λ_i is the eigenvalue $\lambda_1 \geq \lambda_2 \cdots \geq \lambda_K > \lambda_{K+1} \cdots = \lambda_{2M} = \sigma^2$, and $\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_{2M}$ are the corresponding orthonormal eigenvectors. The signal subspace $\boldsymbol{U}_s$ and noise subspace $\boldsymbol{U}_n$ are defined as

$$\boldsymbol{U}_s = [\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_K], \boldsymbol{U}_n = [\boldsymbol{u}_{K+1}, \ldots, \boldsymbol{u}_{2M}] \tag{7}$$

The traditional MUSIC spectrum estimation formula is:

$$P_{MUSIC}(\theta,\varphi,\gamma,\eta) = \frac{1}{\boldsymbol{A}^{\mathrm{H}}(\theta,\varphi,\gamma,\eta)\boldsymbol{U}_n\boldsymbol{U}_n^{\mathrm{H}}\boldsymbol{A}(\theta,\varphi,\gamma,\eta)} \tag{8}$$

To solve equation (8), four-dimensional spectrum peak search is required, which leads to large computation cost. If the problem can be solved by two-dimensional spectrum peak search, the computation will be reduced greatly, which is known as dimension reduction.

The derivation of the new method is as follows

$$T(\theta,\varphi,\gamma,\eta) = \boldsymbol{A}^{\mathrm{H}}(\theta,\varphi,\gamma,\eta)\boldsymbol{U}_n\boldsymbol{U}_n^{\mathrm{H}}\boldsymbol{A}(\theta,\varphi,\gamma,\eta) \tag{9}$$

Because of

$$\begin{aligned} \boldsymbol{A}(\theta,\varphi,\gamma,\eta) &= \boldsymbol{a}_s(\theta,\varphi) \otimes \boldsymbol{a}_p(\theta,\varphi,\gamma,\eta) \\ &= \boldsymbol{a}_s(\theta,\varphi) \otimes \left[\boldsymbol{V}(\theta,\varphi)\cdot \boldsymbol{E}(\gamma,\eta)\right] \end{aligned} \tag{10}$$

$$\boldsymbol{A} \otimes (\boldsymbol{B}\boldsymbol{C}) = (\boldsymbol{A} \otimes \boldsymbol{B})\boldsymbol{C} \tag{11}$$

According to equation (10) and (11), the variant of equation (9) is

$$\begin{aligned} T(\theta,\varphi,\gamma,\eta) &= \boldsymbol{E}^{\mathrm{H}}(\gamma,\eta)\left[\boldsymbol{a}_s(\theta,\varphi)\otimes \boldsymbol{V}(\theta,\varphi)\right]^{\mathrm{H}} \\ &\cdot \boldsymbol{U}_n\boldsymbol{U}_n^{\mathrm{H}}\left[\boldsymbol{a}_s(\theta,\varphi)\otimes \boldsymbol{V}(\theta,\varphi)\right]\boldsymbol{E}(\gamma,\eta) \end{aligned} \tag{12}$$

Define

$$\begin{aligned} &\boldsymbol{G}(\theta,\varphi) = \\ &\left[\boldsymbol{a}_s(\theta,\varphi)\otimes \boldsymbol{V}(\theta,\varphi)\right]^{\mathrm{H}}\boldsymbol{U}_n\boldsymbol{U}_n^{\mathrm{H}}\left[\boldsymbol{a}_s(\theta,\varphi)\otimes \boldsymbol{V}(\theta,\varphi)\right] \end{aligned} \tag{13}$$

According to subspace principle, the eigenvector of the subspace which composed of signal steering vector and the eigenvector of noise subspace are orthogonal.

$$span\{\boldsymbol{A}\} \perp span\{\boldsymbol{U}_n\} \tag{14}$$

Substitute equation (14) into equation (9), yields

$$T(\theta,\varphi,\gamma,\eta) = 0 \tag{15}$$

When $\gamma \in (0, \pi/2)$, $\boldsymbol{a}_p^{\mathrm{H}}(\theta,\varphi,\gamma,\eta)$ has full column rank. To make the equation (15) satisfied, $\boldsymbol{G}(\theta,\varphi)$ is not full rank, so we can include

$$\det\{\boldsymbol{G}(\theta,\varphi)\} = 0 \tag{16}$$

According to equation (17), we can get the DOA estimation:

$$(\hat{\theta},\hat{\varphi}) = \arg\max \frac{1}{\det\{\boldsymbol{G}(\theta,\varphi)\}} \tag{17}$$

Using $T(\theta,\varphi,\gamma,\eta)$ to solve the DOA and polarization parameters estimation can be seen as a solution of the optimization problem. The optimization problem can be described as:

$$\min \boldsymbol{E}^H(\gamma,\eta)\boldsymbol{G}(\theta,\varphi)\boldsymbol{E}(\gamma,\eta) \qquad s.t. \quad \left|\boldsymbol{E}(\gamma,\eta)\right|^2 = 1 \tag{18}$$

The cost equation is established as

$$\begin{aligned} &L(\theta,\varphi,\gamma,\eta) = \\ &\boldsymbol{E}^H(\gamma,\eta)\boldsymbol{G}(\theta,\varphi)\boldsymbol{E}(\gamma,\eta) - u[\boldsymbol{E}^{\mathrm{H}}(\gamma,\eta)\boldsymbol{E}(\gamma,\eta) - 1] \end{aligned} \tag{19}$$

To derivative equation (19) and make the result zero, we have

$$\frac{\partial L(\theta,\varphi,\gamma,\eta)}{\partial \boldsymbol{E}(\gamma,\eta)} = 2\boldsymbol{G}(\theta,\varphi)\boldsymbol{E}(\gamma,\eta) - 2u\boldsymbol{E}(\gamma,\eta) = 0 \tag{20}$$

$$\boldsymbol{G}(\theta,\varphi)\boldsymbol{E}(\gamma,\eta) = u\boldsymbol{E}(\gamma,\eta) \tag{21}$$

Obviously, $\boldsymbol{E}(\gamma,\eta)$ is the eigenvector corresponding to eigenvalue u of $\boldsymbol{G}(\theta,\varphi)$. Due to

$$\boldsymbol{E}^{\mathrm{H}}(\gamma,\eta)\boldsymbol{G}(\theta,\varphi)\boldsymbol{E}(\gamma,\eta) = u\boldsymbol{E}^{\mathrm{H}}(\gamma,\eta)\boldsymbol{E}(\gamma,\eta) = u \tag{22}$$

Therefore, to make the objective function minimum is equal to make u minimum. The desires of $\boldsymbol{E}(\gamma,\eta)$ is just the eigenvector corresponding to minimum eigenvalue of $\boldsymbol{G}(\theta,\varphi)$. That is

$$\boldsymbol{E}(\gamma,\eta) = \boldsymbol{p}_{\min}[\boldsymbol{G}(\theta,\varphi)] \tag{23}$$

According to the expression of $\boldsymbol{E}(\gamma,\eta)$ in equation (1), we can get polarization parameters through following equations

$$\hat{\gamma}_i = \tan^{-1}\{|\xi_i|\} \tag{24}$$

$$\hat{\eta}_i = \arg\{\xi_i\} \tag{25}$$

Where $\xi_i = \frac{\boldsymbol{p}_{\min}[\boldsymbol{G}(\hat{\theta}_i,\hat{\varphi}_i)]_2}{\boldsymbol{p}_{\min}[\boldsymbol{G}(\hat{\theta}_i,\hat{\varphi}_i)]_1}$, $\boldsymbol{p}_{\min}[\boldsymbol{G}(\hat{\theta}_i,\hat{\varphi}_i)]_l$ is the l_{th} element of $\boldsymbol{p}_{\min}[\boldsymbol{G}(\hat{\theta}_i,\hat{\varphi}_i)]$. The DOA estimation and polarization parameter estimation is one to one correspondence, so there is no additional angle matching process.

IV. CALCULATION CONTRAST

Using traditional MUSIC algorithm can obtain DOA estimation and polarization parameter estimation, the calculation is $o\{4M^2L+8M^3+n^4[2M(2M-K)+2M-K]\}$, where n is the total search times within the search range, L is the number of snapshots, M is the number of electric crossed dipole pairs, K is the number of uncorrelated narrowband sources. MC-RDMUSIC algorithm has lower computational complexity than traditional MUSIC algorithm, and it requires $o\{4M^2L+8M^3+n^2[(4M^2+4)(2M-K)+8]\}$. ESPRIT algorithm requires $o\{4M^2L+8M^3+2K^2M+4K^2 \cdot(M-1)+6K^3]\}$. In general, M 、 K and L is much smaller than n, so the computational complexity mainly depends on the items relevant with n. The computational complexity of the traditional MUSIC algorithm is the largest, with the magnitude of n^4. The computational complexity of MC-RDMUSIC algorithm is just of order n^2, so its operation efficiency has significant advantages. The computational complexity of ESPRIT algorithm doesn't include items about n, so it has the smallest calculation complexity. However, ESPRIT algorithm needs to consider parameters matching problem, which leads to large complications.

V. SIMULATIONS

Assuming that the number of array elements is 8, sampling snapshots is 512, the inter-element spacing is set to be $d=\lambda/2$, two incoherent sources $\varphi=\pi/2$, $\eta=\pi/2$, $(\theta_1,\gamma_1)=(10^\circ,30^\circ)$ and $(\theta_2,\gamma_2)=(30^\circ,60^\circ)$ .Change the signal-to-noise-ratio (SNR) of incident signal, and conduct Monte Carlo experiment 500 times. The angle error of the measured value is in the range of 1 degree is defined as success, otherwise for failure. After a large number of experiments, the number of successful experiment gives the success rate. Figure 1(a) and 1(b) show a comparison between the performances of traditional MUSIC algorithm, ESPRIT algorithm and MC-RDMUSIC algorithm. The success rate of DOA and polarization parameters versus SNR is shown in Figure 1(a) and 1(b). It can be seen that success rate of DOA and polarization parameters estimation is increasing as the SNR increases. In the case of low SNR, success rate of traditional MUSIC algorithm and MC-RDMUSIC algorithm is slightly higher than ESPRIT algorithm.

The root mean square error (RMSE) of DOA and polarization parameters versus SNR under three algorithms is shown in Figure 2(a) and 2(b) respectively. From the simulation results we can see that as the SNR increases, estimation error is smaller and smaller. For DOA measuring accuracy, traditional MUSIC algorithm and MC-RDMUSIC algorithm is nearly the same and higher than ESPRIT algorithm. For polarization parameters measuring accuracy, In the case of low SNR, traditional MUSIC algorithm is higher than MC-RDMUSIC algorithm and ESPRIT algorithm.

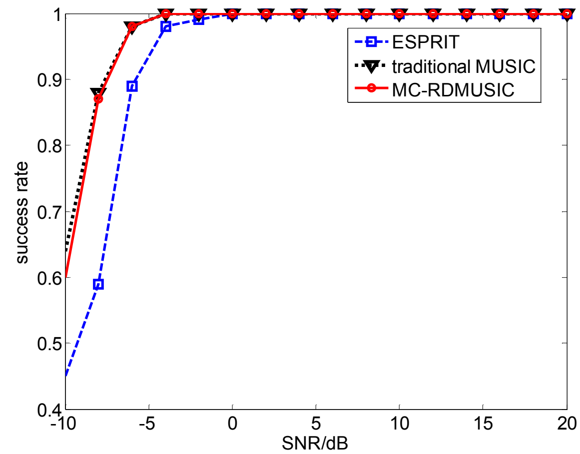

Fig. 1(a).success rate of DOA estimation versus SNR

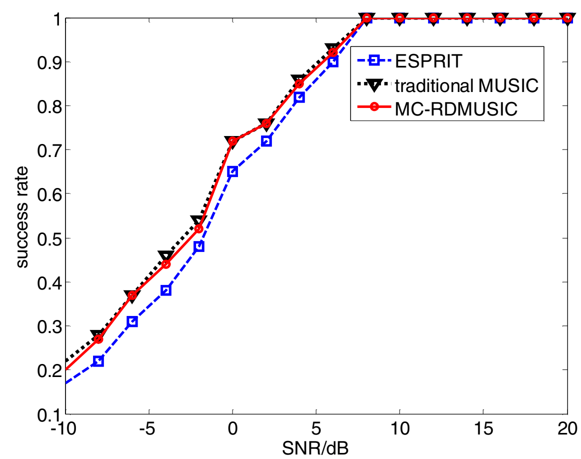

Fig. 1(b).success rate of polarization parameters versus SNR

Assuming SNR is $10dB$, change the number of sampling snapshots. The root mean square error (RMSE) of DOA and polarization parameters versus sampling snapshots under three algorithms is shown in Figure 3(a) and 3(b) respectively. From the simulation results it can be seen, as the increase of sampling snapshots, RMSE is degrading consistently. Under the condition of small number of snapshots, traditional MUSIC algorithm and MC-RDMUSIC algorithm has the higher precision than ESPRIT algorithm for DOA estimation and polarization parameters estimation. However, increasing snapshots number will lead to the increase of computation complexity, we should choose appropriate sampling snapshots. In this experiment, when the snapshots number arrives 300, the RMSE of DOA estimation and polarization parameters is already small and nearly stable.

Therefore, we can conclude that the performance of MC-RDMUSIC is very close to traditional MUSIC algorithm for DOA estimation, and better than ESPRIT algorithm. For polarization parameters estimation, the performance of MC-RDMUSIC is slightly inferior to traditional MUSIC algorithm, but better than ESPRIT algorithm.

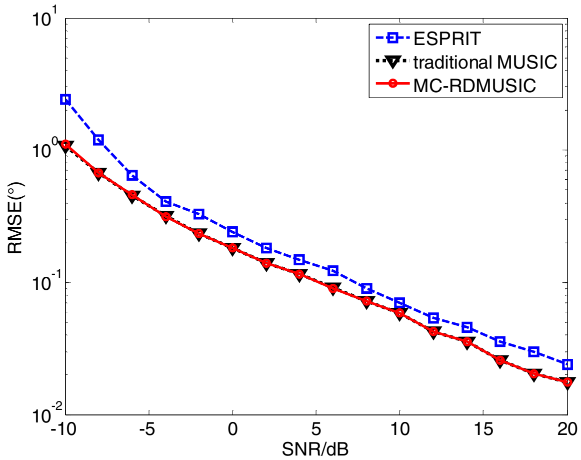

Fig. 2(a).RMSE of DOA estimation versus SNR

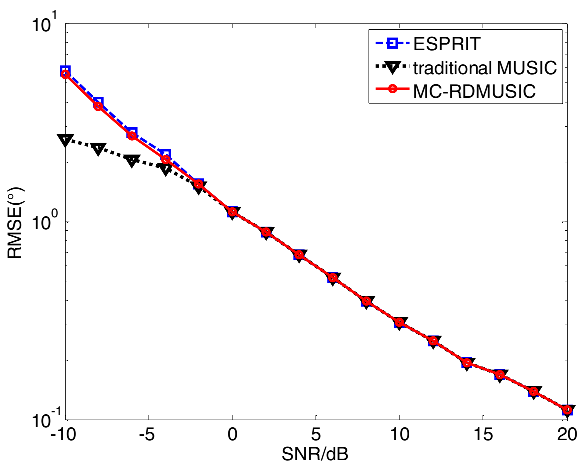

Fig. 2(b).RMSE of polarization parameters versus SNR

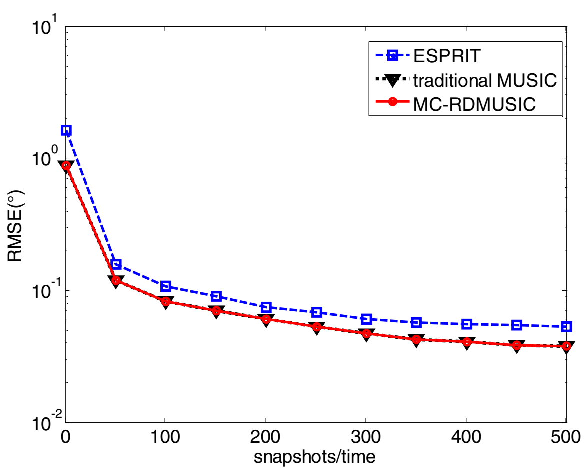

Fig. 3(a).RMSE of DOA estimation versus snapshots

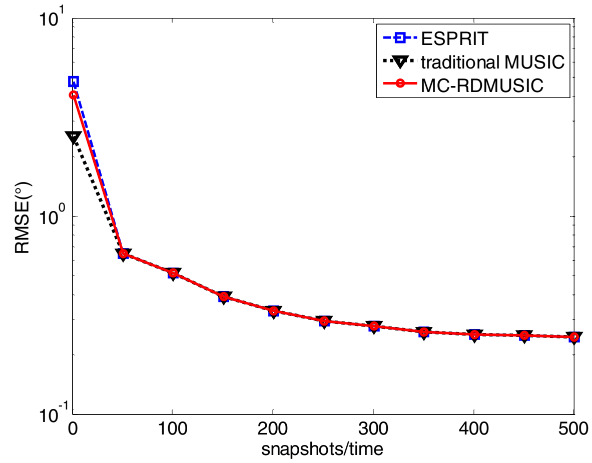

Fig. 3(b).RMSE of polarization parameters versus snapshots

VI. CONCLUSION

In order to reduce computational complexity and avoid parameters matching problem, MC-RDMUSIC algorithm is proposed. Direction-of-arrival angle information is separated from polarization information and estimated by two-dimensional MUSIC algorithm, and then closed-form formulas are derived and utilized to obtain polarization parameter estimation. The simulation experiment results verify the accuracy of reduced-dimensional MUSIC algorithm with modulus constraint. Compared with traditional MUSIC algorithm and ESPRIT algorithm, MC-RDMUSIC has an acceptable estimation precision and success rate performance, while the computation complexity is highly reduced. Besides that, on the basis of the algorithm introduced in this paper, we will in the future consider a new method that completely avoid spectrum peak search, further reduce the computation.

VII. REFERENCES

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